K. E. W.

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THE PROJECTION OF THE SPHERE,

ORTHOGRAPHIC, STEREOGRAPHIC, and GNOMONICAL.

Both demonſtrating the

PRINCIPLES,

And explaining the

PRACTICE

Of theſe three ſeveral Sorts of PROJECTION.

The SECOND EDITION, Corrected and Improved.

In Minimis Uſus——

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THE PREFACE.

THE Projection of the Sphere, or of its Circles, has the same relation to Spherical Trigonometry, that practical Geometry has to plane Trigonometry. For as the one saves a deal of Calculation, by drawing a few right Lines, so does the other by drawing a few Circles. The Projection of the Sphere gives a Learner a good Idea of the Sphere and all its Circles, and of their several Positions to one another, and consequently of Spherical Triangles, and the Nature of Spherical Trigonometry.

I have here delivered the Principles of three sorts of Projection, in a small compass; and yet the Reader will find here, all that is essential to the subject; and yet nothing superfluous; for I think no more need be said, or indeed can be said about it, to make it intelligible and practicable. For here is laid down, not only the whole Theory, but the Practice likewise. Yet the practical Part is entirely disengaged from the Theory; so that any body (tho' he has no desire or leisure to attain to the Theory,) may nevertheless, by help of the Problems, make himself Master of the Practice. For which end I have endeavoured to make all the rules relating to practice, plain, short, and easy, and at the same time full and clear.

It is true the solution of Problems this way, must be allowed to be imperfect; for there will always be some errors in working, as well as in the instruments

we work with. But nobody in ſeeking an accurate ſolution to a Problem, will truſt to a Projection by ſcale and compaſs; becauſe this cannot be depended on in caſes of great nicety. Yet where no great exactneſs is required it will be found very ready and uſeful; and, beſides, will ſerve to prove and confirm the ſolution obtaind by Calculation.

But then this defect is abundantly recompenſed by the eaſineſs of this method. For by ſcale and compaſs only, all ſorts of Problems belonging to the Sphere, as in Aſtronomy, Geography, Dialling, &c. may be ſolved with very little trouble, which require a great deal of time and pains, to work out trigonometrically by the tables. It likewiſe affords a great pleaſure to the mind, that one can, in a little time, deſcribe the whole furniture of Heaven, and Earth, and repreſent them to the eye, in a ſmall ſcheme of paper.

But its principal uſe is for ſuch perſons (and that is by far the greater number) as having no opportunity for learning Spherical Trigonometry, have yet a deſire to reſolve ſome Problems of the Sphere. For ſuch as theſe, this ſmall Treatiſe will be of particular ſervice, becauſe the practical rules, eſpecially of any one ſort of Projection, may be learned in a very little time, and are eaſily remembered. So that I have ſome hopes I ſhall pleaſe all my Readers, whether theoretical or practical.

W. E.

THE

THE PROJECTION OF THE SPHERE IN PLANO.

DEFINITIONS.

1. PROJECTION of the sphere is the representing its surface upon a plane, called the *Plane of Projection*.

2. *Orthographic* Projection, is the drawing the circles of the sphere upon the plane of some great circle, by lines perpendicular to that plane, let fall from all the points of the circles to be projected.

3. The *Stereographic* Projection, is the drawing the circles of the sphere upon the plane of one of its great circles, by lines drawn from the pole of that great circle to all the points of the circles to be projected.

4. The *Gnomonical* Projection, is the drawing the circles of an hemisphere, upon a plane touching it in the vertex, by lines or rays issuing from the center of the hemisphere, to all the points of the circles to be projected.

5. The *Primitive* circle is that on whose plane the sphere is projected. And the pole of this circle is called the *Pole* of Projection. The point from whence the *projecting* right lines issue is the *projecting Point*.

6. The *Line of Meaſures* of any circle is the common interſection of the plane of projection, and another plane that paſſes thro' the eye, and is perpendicular both to the plane of projection, and to the plane of that circle.

SCHOLIUM.

There are other Projections of the Sphere, as the *Cylindrical*, the *Scenographic* which belongs to Perſpective, the *Globical* which belongs to Geography, *Mercators*, for which ſee Navigation, &c.

AXIOM.

The *Place* of any viſible point of the Sphere upon the plane of projection, is where the projecting line cuts that plane.

Cor. *If the eye be applied to the projecting point, it will view all the circles of the Sphere, and every part of them, in the projection, juſt as they appear from thence in the Sphere itſelf.*

SCHOLIUM.

The Projection of the Sphere is only the ſhadow of the circles of the Sphere upon the plane of Projection, the light being in the place of the eye or projecting point.

The Signification of ſome Characters.

$+$ added to.
$-$ ſubtracting the following quantity.
$<$ an angle.
$=$ equal to.
$\perp$ perpendicular to.
$\parallel$ parallel to.
$::$ a proportion.

SECT.

SECT. I.

The Orthographic Projection of the SPHERE.

PROP. I.

IF a right line AB *is projected upon a plane, it is projected into a right line; and its length will be to the length of the projection, as radius to the cosine of its inclination above that plane.* Fig. 3.

For let fall the perpendiculars A*a*, B*b* upon the plane of projection, then *ab* will be the line it is projected into; but by trigonometry AB : is to A*o* or *ab* : : as radius : to the sine of B or cosine of *o*AB.

Cor. 1. *If a right line is projected upon a plane, parallel thereto, it is projected into a right line parallel and equal to itself.*

Cor. 2. *If an angle be projected upon a plane which is parallel to the two lines forming the angle; it is projected into an angle equal to itself.*

Cor. 3. *Any plain figure projected upon a plane parallel to itself, is projected into a figure similar and equal to itself.*

Cor. 4. *Hence also the area of any plain figure, is to the area of its projection : : as radius, to the cosine of its elevation or inclination.*

PROP. II.

A circle perpendicular to the plane of projection, is projected into a right line equal to its diameter.

For projecting lines drawn through all the points of the circle fall in the common section of the planes

 of

Fig. of the circle and of projection, which is a right line (Geom. V. 3.), and equal to the diameter of the circle; becauſe the planes interſect in that diameter. *Q. E. D.*

Cor. *Hence any plane figure, perpendicular to the plane of projection is projected into a right line. For the perpendiculars from every point, will all fall in the common interſection of the figure with the plane of projection.*

PROP. III.

1. *A circle parallel to the plane of projection is projected into a circle equal to itſelf, and concentric with the primitive.*

Let BOD be the circle, I its center, C the center of the ſphere, the points I, B, O, D, are projected into the points C, L, F, G. And therefore OICF, and BICL are rectangled parallelograms. Conſequently LC = BI = OI = FC, (Geom. III. 1.). *Q. E. D.*

Cor. *The radius* CL *or* CF *is the coſine of the circle's diſtance from the primitive, for it is the ſine of* AB.

PROP. IV.

2. *An inclined circle is projected into an ellipſis whoſe tranſverſe axis is the diameter of the circle.*

Let ADBH be the inclined circle, P its center; and let it be projected into *adbh*; draw the plane ABFC*a* through the center C of the ſphere, perpendicular to the plane of the given circle and plane of projection, to interſect them in the lines AB, *ab*; draw GPH, DE, perpendicular, and DQ parallel to AB; then becauſe the line GP, and the plane of projection are both perpendicular to the

the plane ABF; therefore GH is parallel to the plane of projection, and therefore to gh. Fig.

In the circle ADB, $DQ^2 = GQH = gqh$, and and $BP^2 = GP^2 = gp^2$. And (Geom. V. 12.) BP : EP or DQ : : $bp : ep$ or dq, and $BP^2 : DQ^2 : : bp^2 : dq^2$; that is, $gp^2 : gqh : : bp^2 : dq^2$; and therefore $agbh$ is an ellipsis, whose transverse gh is the diameter of the circle. *Q. E. D.*

Cor. 1. *Since ab is perpendicular to gh, therefore ab is the conjugate axis; and is twice the sine of the* $<$ AB*b to the radius gp; that is, the conjugate axis is equal to twice the cosine of the inclination, to the radius of the circle.*

Cor. 2. *The transverse axis is equal to twice the cosine of its distance from its parallel great circle. For gh* = GH = 2AP = *twice the sine of* AK.

Cor. 3. *The extremities of the conjugate axis are distant from the center of the primitive, by the sines of the circles nearest and greatest distance from the pole of the primitive. Thus a*C *is the sine of* AN, *and b*C *the sine of* BN.

Cor. 4. *Hence also it is plain that the conjugate axis always passes thro' the center* C *of the primitive; and is always in the line of measures of that circle.*

Scholium.

Every circle in the projection represents two equal circles, parallel and equidistant from the primitive. Every right line represents two semicircles, one towards the eye, the other in the opposite side. Every ellipsis represents two equal circles, but contrarily inclined as AB, CD; one above the primitive the other below it. 3.

And now the Theory being laid down, it remains only to deduce thence, some short rules for practice, as follows.

PROP.

Fig.

PROP. V. *Prob.*

5. *To project a circle parallel to the primitive.*

Rule.

Take the complement of its diſtance from the primitive, and ſet it from A to E; and with the center C and radius CD = perpendicular EF, deſcribe the circle DgG.

By the plain ſcale.

Take the ſine of its diſtance from the pole of the primitive; with that radius and the center C deſcribe the circle.

PROP. VI. *Prob.*

4. *To project a right circle, or one that is perpendicular to the plain of projection.*

Rule.

Thro' the center C of the primitive, draw the diameter AB, and take the diſtance from its parallel great circle, and ſet from A to E, and from B to D, and draw ED, the right circle required.

By the ſcale.

Take the ſine of the circle's diſtance from its parallel great circle AB, and at that diſtance draw a parallel ED for the circle required.

PROP. VII. *Prob.*

To project a given oblique circle.

Rule.

6. Draw the line of meaſures AB, and take the circle's neareſt diſtance from the primitive, and ſet from

from B to D, upwards if it be above the primitive; Fig.
or downward, if below; likewise take its greatest 6.
distance, and set from A to E, and draw ED, and let fall the perpendiculars EF, DG; and bisect FG in H, and erect the perpendicular KHI, making KH = HI = half ED; then describe an ellipsis (by the Conic Sections) whose transverse is IK and conjugate FG; and that shall represent the circle given.

By the scale.

Draw the line of measures AB; and take the 6.
sines of the circle's nearest and greatest distance from the pole of the primitive, and set them from the center C to F and G, (both ways if the circle encompass the pole, but the same way if it lie on one side the pole;) bisect FG in H, and erect HK, HI perpendicular to FG, and = to the radius of the circle given, or the sine of its distance from its own pole; about the axes FG, KI describe an ellipsis, and it is done.

SCHOLIUM.

An ellipsis great or small may be described by 10.
points, thus; thro' the center D of the circle and ellipsis, draw BD ⊥ the transverse AR; and on AR erect a sufficient number of perpendiculars IK, *ik* &c. and make as DB or DA : DE : : IK : IF : : *ik* : *if* &c. then thro' all the points E, F, *f*, &c. draw a curve. See Prop. 76. ellipsis.

PROP. VIII. *Prob.*

To find the pole of a given ellipsis.

Rule.

Thro' the center of the primitive C, draw the 7.
conjugate of the ellipsis; on the extreme points F, G, erect the perpendiculars FE, GD, or set the transverse

Fig. tranſverſe IK from E to D, and biſect ED in R,
7. and let fall RP perpendicular to AB, then is P the pole.

By the ſcale.

7. Take CF, and CG, and apply to the ſines, and find the degrees anſwering or the ſupplements; then take the ſine of half the ſum of theſe degrees, if F, G be both on one ſide of C, or the ſine of half the difference, if they lie on contrary ſides; and ſet it from C to the pole P.

Or thus; apply the ſemi-tranſverſe IH to the ſines, and ſet the degrees from E to R; and draw RP ⊥ to AB; and P is the pole.

PROP. IX. *Prob.*

To meaſure an arch of a parallel circle; or to ſet any number of degrees on it.

Rule.

With the radius of the parallel, and one foot in C deſcribe a circle G*g*, draw CGB, and C*gb*; then B*b* will meaſure the given arch G*g*; or G*g* will contain the given number of degrees ſet from B to *b*. So that either being given finds the other.

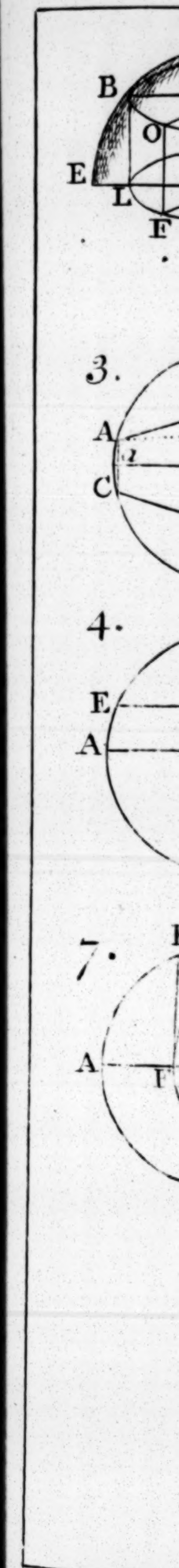
PROP. X. *Prob.*

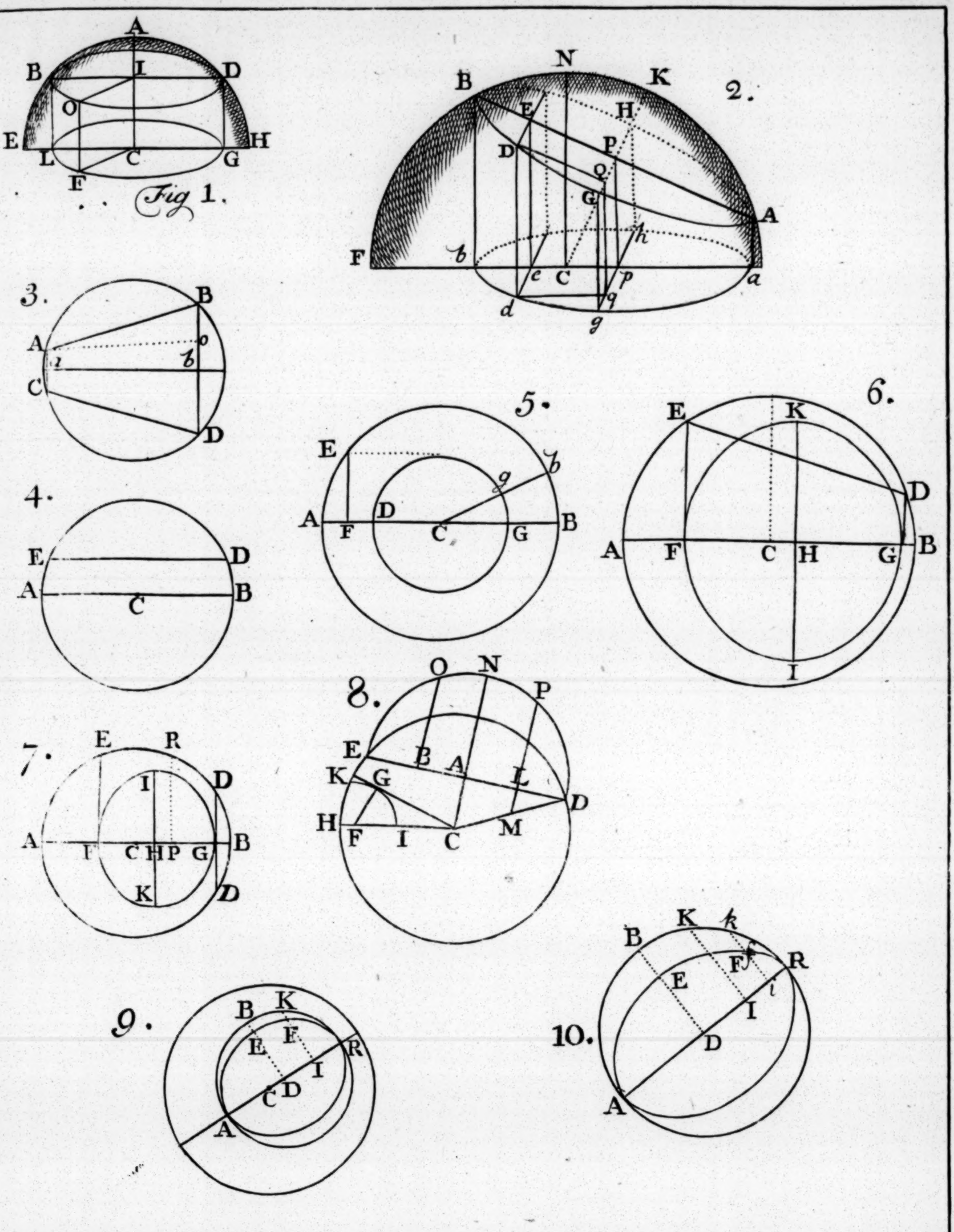
To meaſure any part of a right circle.

Rule.

8. In the right circle ED, let EA = AD; and let AB be to be meaſured. Make CF = AE; with extent BA = FG deſcribe the arch GI; draw CGK to touch it in G; then is HK the meaſure of AB. For FG = S. ∠ HCK to the radius CF or AE, and BA is the ſame, by Cor. Prop. III.

Other-





rojection.

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Fig. 8.

Otherwise thus.

On the diameter ED, describe the semicircle END, draw AN, BO, LP perpendicular to ED, then ON is the measure of BA, and NP of AL; and ON or NP may be measured as in Prop. IX.

By the scale.

Let AL be to be measured. Draw CD; and LM parallel to AC, then CM applied to the sines gives the degrees. For radius CD : AD : : CM : AL.

Cor. *If the right circle passes thro' the center, there is no more to do, but to raise perpendiculars on it, which will cut the primitive, as required. Or apply the part of the right circle to the line of sines.*

PROP. XI. *Prob.*

To set off any number of degrees upon a right circle, DE.

Rule.

Draw CA $\perp$ DE, and make the $<$ HCK $=$ the degrees given, make CF $=$ radius AE, take FG the nearest distance, and set from A to B; then AB $= <$ HCK, the degrees proposed. 8.

Otherwise thus.

On ED describe the semicircle END, then by Prop. IX. set off NP $=$ degrees given, draw PL perpendicular to ED, then AL contains the degrees required.

Or thus by the scale.

Draw CD, take the given degrees of the sines, and set from C to M, and draw ML parallel to CA, then AL $=$ arch required.

PROP.

Fig.

PROP. XII. *Prob.*

9. *To measure an arch of an ellipsis; or to set any*
10. *number of degrees upon it.*

Rule.

About AR the transverse axis of the ellipsis, describe a circle ABR; erect the perpendiculars BED, KFI, on AR; then BK is the measure of EF, or EF is the representation of the arch BK. And BK is to be measured, or any degrees set upon it, as in Prop. IX.

Scholium.

These Problems are all evident from the three first propositions, and need no other demonstration. If the sphere be projected on any plane parallel to the primitive, the projection will be the very same; for being effected by parallel lines, which are always at the same distance, there will be produced the same figure, or representation. Of all orthographic projections, those on the meridian or on the solstitial colure, commonly called the Analemma, is most useful; because a great many of the circles of the sphere fall into right lines or circles, whereas in the projections upon other planes, they are projected into ellipses, which are hard to describe; which makes these sorts of projection to be neglected.

And by the same rules that the circles of the sphere are projected upon a plane, any other figure may likewise be orthographically projected; by letting fall perpendiculars upon the plane from all the angles, or all the points of the figure, and joining these points with right or curve lines, as they are in the figure itself.

By this kind of projection, either the convex or concave side of the sphere, may be projected; which

which is peculiar to this sort of projection; that Fig. 9.
is, either the hemisphere towards you, or that from you, may be projected upon the plane of its great circle. And since in some cases they both have the same appearance, it ought to be mentioned whether it is.

But if both the convex and concave sides of the *same hemisphere* be projected; that is, if you make two projections, one for the convex, the other for the concave side; the circles in one will be inverted in respect of the other, the right to the left, &c. Because in looking at the same hemisphere, it will not have the same appearance, when you look at the contrary sides of it; because you look contrary ways at it, to see the external and internal surfaces.

SECT.

Fig.

SECT. II.

The Stereographic Projection of the SPHERE.

PROP. I.

ANY circle passing thro' the projecting point, is projected into a right line.

For all lines drawn from the projecting point, to this circle, pass thro' the intersection of this circle and plane of projection, which is a right line.

Cor. 1. *A great circle passing thro' the poles of the primitive is projected into a right line passing thro' the center.*

Cor. 2. *Any circle passing thro' the projecting point is projected into a right line perpendicular to the line of measures, and distant from the center, the semitangent of its nearest distance from the pole opposite to*
12. *the projecting point. Thus the circle* AE *is projected into a right line passing thro'* G, *and perpendicular to* BC, *the line of measures, and* GC *is the semitangent of* EM.

PROP. II.

Every circle (that passes not through the projecting point) is projected into a circle.

11. *Case* I. Let the circle EF be parallel to the primitive BD; lines drawn to all points of it from the projecting point A, will form a conic surface, which being cut parallel to the base by the plane BD, the section GH (into which EF is projected) will be a circle by the conic sections.

Case

Case II. Let BH be the line of measures to the circle EF, draw FK parallel to BD, then arch AK = AF, and therefore < AFK or AHG = AEF; therefore in the triangles AEF, AGH, the angles at E and H are equal, and the angle A common; therefore the angles at F and G are equal. Therefore the cone of rays AEF (whose base EF is a circle) is cut by subcontrary section, by the plane of projection BD, and therefore, by the conic sections, the section GH (which is the projection of the circle EF) will also be a circle. *Q. E. D.* Fig. 12.

Cor. *When* AF *is equal to* AG, *the circle* EF *is projected into a circle equal to it self.*

For then the similar triangles AHG and AEF, will also be equal, and GH = EF.

PROP. III.

Any point on the sphere's surface is projected into a point, distant from the center, the semi-tangent of its distance from the pole opposite to the projecting point.

Thus the point E is projected into G, and F into H; and CG is the semi-tangent of EM, and CH of MF.

Cor. 1 *A great circle perpendicular to the primitive is projected into a line of semi-tangents passing thro' the center, and produced infinitely.*

For MF is projected into CH its semi-tangent, and EM into the semi-tangent CG.

Cor. 2. *Any arch* EM *of a great circle perp. to the primitive; is projected into the semi-tangent of it.*

Thus EM is projected into GC.

Cor. 3. *Any arch* EMF *of a great circle is projected into the sum of its semi-tangents, of its greatest*

Fig. 12. *and least distances from the opposite pole* M, *if it lye on both sides of* M, *or the dif. of the semi-tangents, when all on one side.*

PROP. IV.

13. *The angle made by two circles on the surface of the sphere is equal to that made by their representatives upon the plane of projection.*

Let the angle BPK be projected. Thro' the angular point P and the center C, draw the plane of a great circle PED perpendicular to the plane of projection EFG. Let a plane PHG touch the sphere in P; then since the circle EPD is perpendicular both to this plane and to the plane of projection, therefore it is perpendicular to their intersection GH. The angles made by circles are the same as those made by their tangents, therefore in the plane PGH, draw the tangents PH, PF, PG to the arches, PB, PD, PK; and these will be projected into the lines pH, pF, pK: Now I say the $<$ HPG $= <$ HpG. For the angle CPF = a right angle = CpA + CAP; therefore taking away the equal angles CPA and CAP, and $<p$PF = CpA or PpF; consequently pF = PF. Therefore in the right angled triangles PFG and pFG, there are two sides equal and the included $<$ right; therefore hypothenuse PG $= p$G. And for the same reason in the right angled triangles PFH and pFH, PH $= p$H. Lastly in the triangles PHG and pHG, all the sides are respectively equal, and therefore $<$ P $= < p$. *Q. E. D.*

Cor. 1. *The rumb lines projected make the same angles with the meridians as upon the globe; and therefore are logarithmic spirals on the plain of the equinoctial. For every particle of the rumb coincides with some great circle.*

Cor.

Cor. 2. *The angle made by two circles on the sphere* Fig.
is equal to the angle made by the radii of their projections at the point of intersection. For the angle made by two circles on a plane is the same with that made by their radii drawn to the point of intersection.

PROP. V.

The center of a projected (lesser) circle perpendicular to the primitive, is in the line of measures distant from the center of the primitive, the secant of the lesser circles distance from its own pole; and its radius is the tangent of that distance.

Let A be the projecting point, EF the circle to 14.
be projected, GH the projected diameter. From the centers C, D draw CF, DF, and the triangles CFI, DFI are right angled at I; then $< IFC = < FCA = 2 < FEA$ or $2FEG = 2 < FHG = < FDG$, therefore $IFC + IFD = FDG + IFD$ = a right angle; that is CFD is a right angle, and the line CD is the secant of BF, and the radius FD is the tangent of it. *Q. E. D.*

Cor. *If these circles be actually described, 'tis plain the radius* FD *is a tangent to the primitive at* F, *where the lesser circle cuts it.*

PROP. VI.

The center of Projection of a great circle is in the line of measures, distant from the center of the primitive, the tangent of its inclination to the primitive; and its radius is the secant of its inclination.

Let A be the projecting point, EF the great circle, 15.
GH the projected diameter, D the center; draw

Fig. draw DA. The angle EAF being in a ſemicir-
15. cle is right. In the right angled triangle GAH, AC is perpendicular to GH, therefore $<$ GAC $=$ AHC and their double, ECB $=$ ADC, and their complements. ECF $=$ CAD. Therefore CD is the tangent of ECI, and radius AD its ſecant. *Q. E. D.*

16. Cor. 1. *If the great oblique circle* AGBH *be actually deſcribed upon the primitive* AIB. *I ſay, all great circles paſſing thro'* G *will have the centers of their projections in the line* RS *drawn thro' the center* D, *perpendicular to the line of meaſures* IH.

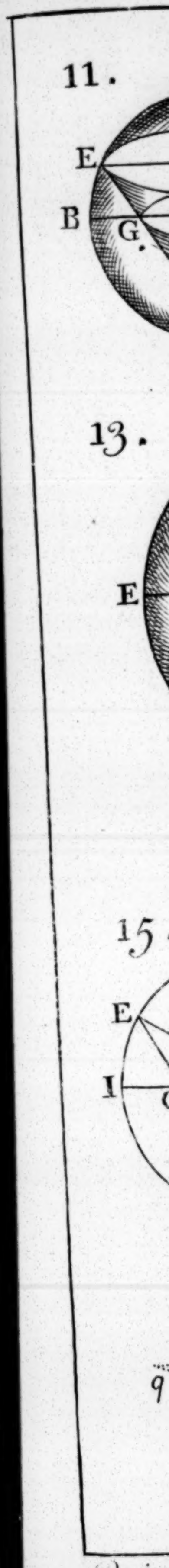
For ſince all great circles cut one another at a ſemicircle's diſtance, all circles paſſing thro' G muſt cut at the oppoſite point H; and therefore their centers muſt be in the Line RDS.

Cor. 2. *Hence alſo if any oblique circle* GLH *be required to make any given angle with another circle* BGAH, *it will be projected the ſame way with regard to* GAH *conſidered as a primitive, and* RS *its line of meaſures; as the circle* BGA *is on the primitive* BIA, *and line of meaſures* ID. *And therefore the tangent of the angle* AGL *to the radius* GD, *ſet from* D *to* N, *gives the center of* GL.

For the $<$ NGD will then be equal to AGL, by Cor. 2. Prop. IV. and therefore GLH is rightly projected.

Cor. 3. *And for the ſame reaſon, if* N *be the center of the circle* GgHR; *the centers of all circles paſſing thro'* g *and* R, *will be in the line* rNs *perpendicular to* RS; *ſo* n *is the center of* grR. *But then as* g, R *do not repreſent oppoſite points of the circle* GgH, *therefore all circles paſſing thro'* g, R, (*as* grR) *will be leſſer circles, except* GgHR.

SCHOLIUM.



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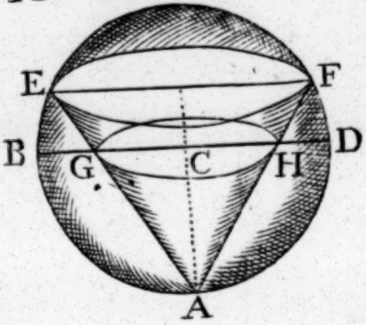

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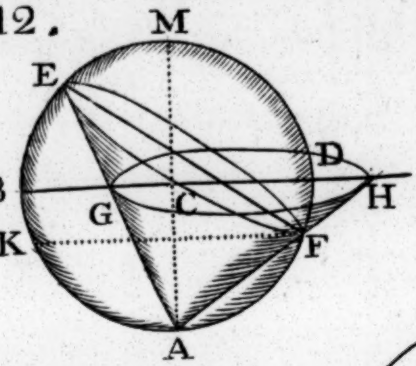

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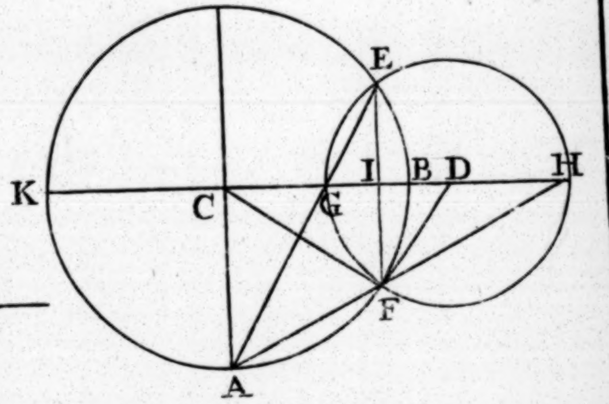

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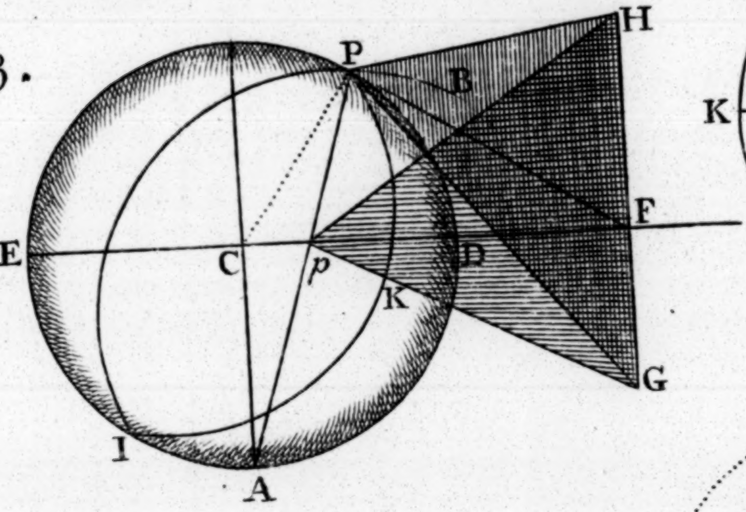

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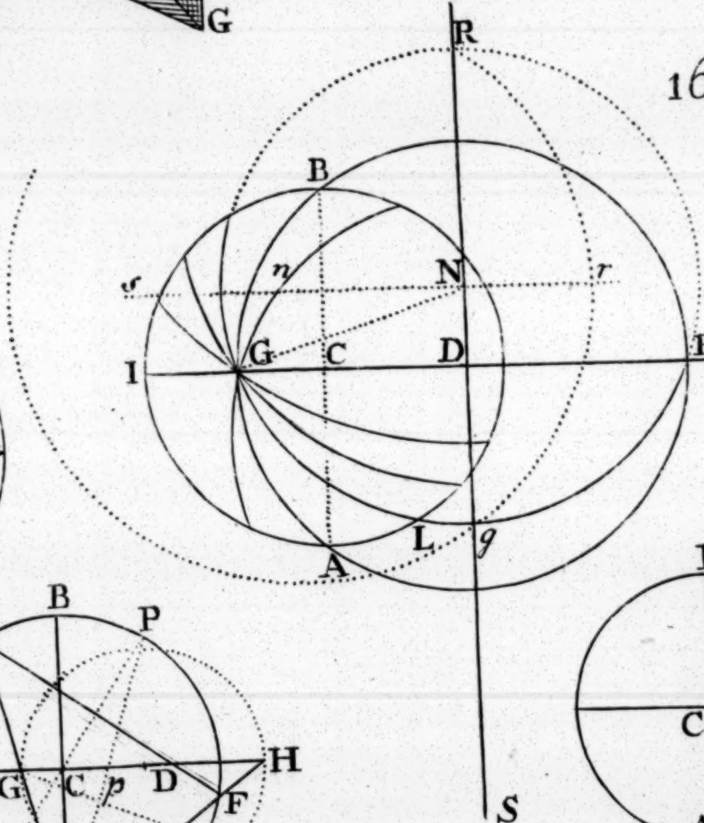

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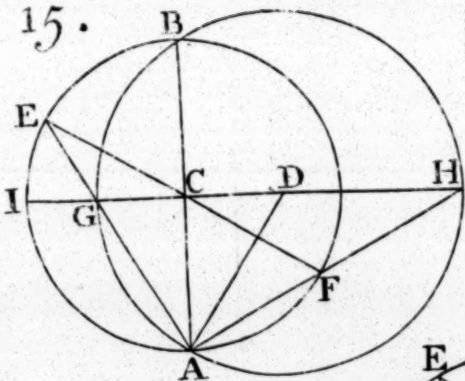

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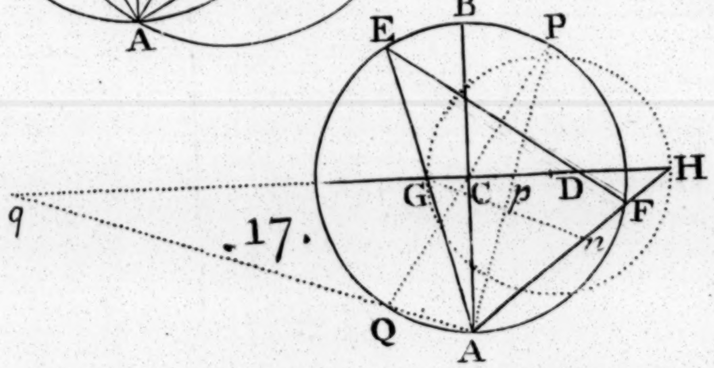
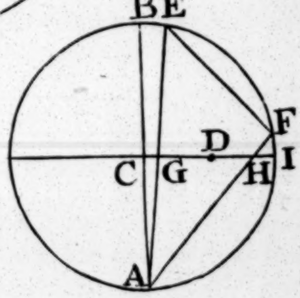




18.

SCHOLIUM. Fig. 16.

Of all great circles in the projection, the primitive is the leaſt. For the radius of any oblique great circle (being the ſecant of the inclination) is greater than the radius of the primitive; as the ſecant is always greater than the radius. Therefore every oblique great circle in the projection is greater than the primitive.

PROP. VII.

The projected extremities of the diameter of any circle, are in the line of meaſures, diſtant from the center of the primitive circle, the ſemi-tangents of its neareſt and greateſt diſtances from the pole of projection oppoſite to the projecting point.

For the diameter of the circle EF is projected 15.
into GH, from the projecting point A. But GC 17.
is the ſemi-tangent of EB, and CH the ſemi-tangent of BF. *Q. E. D.*

Cor. 1. *The points where an inclined great circle* 15.
cuts the line of meaſures, within and without the primitive, is diſtant from the center of the primitive, the tangent and co-tangent of half the complement of the circle's inclination to the primitive

For CG = tangent of half EB, or of half the complement of IE the inclination. And (becauſe the $\angle$ EAF is right) CH is the co-tangent of GAC or half EB.

Cor. 2. *Hence the center* D *of a projected circle is* 17.
in the line of meaſures diſtant, from the center of the 18.
primitive, half the difference of the ſemi-tangents of its neareſt and greateſt diſtance from the oppoſite pole, if it encompaſſes that pole; but half the ſum of the ſemi-tangents if it lye on one ſide the pole of projection.

Fig. Cor. 3. *And the radius is half the ſum of the ſemi-tangents, if the circle encompaſſes the pole; or half the difference if it lyes on one ſide.*

17. Cor. 4. *Hence alſo if pq be the projected poles, it will be* $qG : pG :: qH : pH$.

For draw Gn parallel to qA, and ſince P, Q are the poles, therefore qAp is a right angle, and ſince the angles GAp and pAH are equal, and Gn perpendicular to Ap, therefore $GA = An$; whence by ſimilar triangles $qG : qH :: An$ or $AG : AH :: Gp, pH$, (Geom. II 25.) And conſequently the line qH is cut horomonically in the points G, p.

PROP. VIII.

The projected poles of any circle are in the line of meaſures, within and without the primitive, and diſtant from its center the tangent and co-tangent of half its inclination to the primitive.

19. The poles P, p of the circle EF are projected into D and d; and CD is the tangent of CAD or half BCP, that is, of half GCI, the inclination of the circle ICK, parallel to EF. Likewiſe Cd is the tangent of CAd, or the co-tangent of CAD.

Q. E. D.

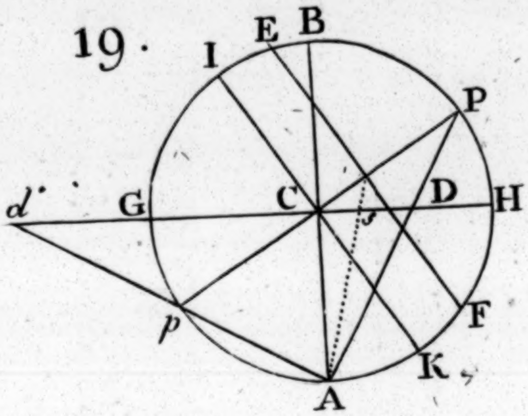
Cor. 1. *The pole of the primitive is its center; and the pole of a right circle is in the primitive.*

19. Cor. 2. *The projected center of any circle is always between the projected pole (neareſt to it on the ſphere) and the center of the primitive; and the projected centers of all circles lye between the projected poles.*

For the middle point of EF or its center is projected into S; and all the points in Pp (in which are all the centers) are projected into Dd.

Cor.

19.



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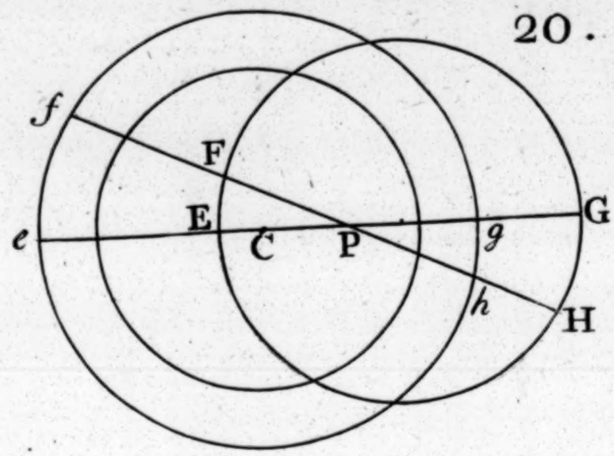

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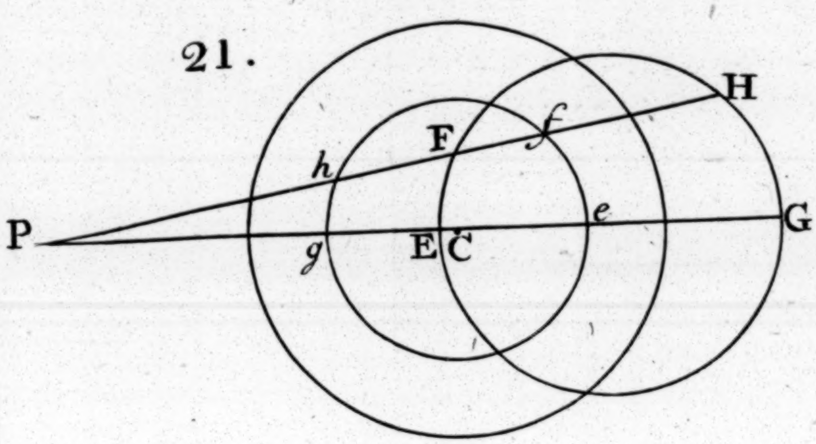

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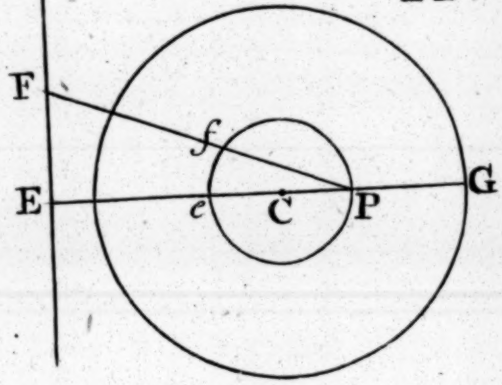

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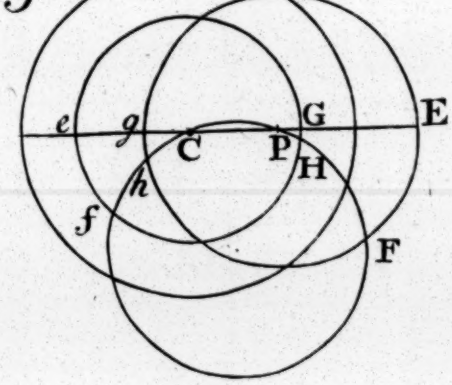

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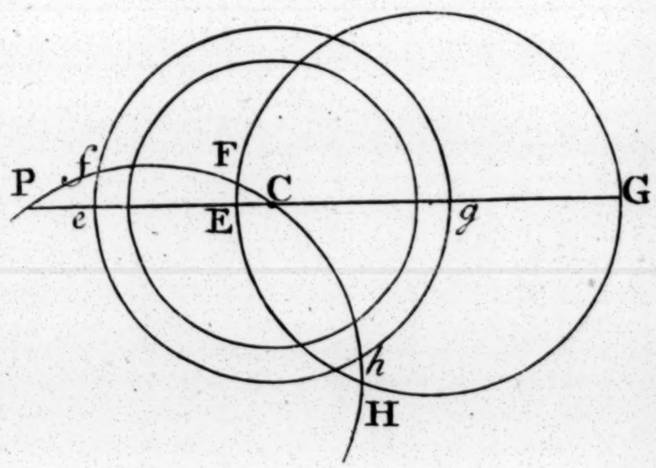

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Cor. 3. *If* P *be the projected center of any circle* Fig.
EFG, *any right lines* EG, FH *passing thro'* P *will* 20.
intercept equal arches EF, GH.

For in any circle of the sphere, any two lines, passing thro' the center, intercept equal arches; and these are projected into right lines, passing thro' the projected center P, and therefore EF, GH, represent equal arches.

PROP. IX.

If EFGH, *efgh represent two equal circles, where-* 20.
of EFG *is as far distant from its pole* P, *as efg is* 21.
from the projecting point. I say, any two right lines (*e*EP, *and f*FP,) *being drawn thro'* P, *will intercept equal arches* (*in representation*) *of these circles; on the same side, if* P *falls within the circles; but on the contrary side, if without; that is,* EF $=$ *ef, and* GH $=$ *gh*.

For by the nature of the section of a sphere; any two circles passing thro' two given points or poles on the surface of the sphere, will intercept equal arches of two other circles equidistant from these poles. Therefore the circles EFG and *efg* on the sphere, are equally cut by the planes of any two circles passing thro' the projecting point and the pole P, on the sphere. But these circles (by Prop. I.) are projected into the right lines P*e* and P*f*, passing thro' P. And the intercepted arches representing equal arches on the sphere, are therefore equal, that is, EF $=$ *ef*, and GH $=$ *gh*.

Cor. 1. *If a circle is projected into a right line* EF, 22.
perpendicular to the line of measures EG; *and if from the center* C *a circle ef*P *be described passing thro' its pole* P, *and* P*f be drawn; then arch ef* $=$ EF. *And if any other circle be described whose vertex is* P, *the arch ef will always be equal to* EF.

Fig. Cor. 2. *Hence alſo, if from the pole of a great circle there be drawn two right lines, the intercepted arch of the projected great circle will be equal to the intercepted arch of the primitive.*

23. Cor. 3. *After the ſame manner, if there be two*
24. *equal circles* EF, *ef, whereof one is as far from the pole* P, *as the other is from the pole of projection e, oppoſite to the projecting point. Then any circle drawn thro' the points* P, C, *will intercept equal arch* EF $=$ *ef*; *and* GH $=$ *gh, between it and the line of meaſures* PCG.

For this is true on the ſphere, and their projections are the ſame.

Cor. 4. *If from an angular point be drawn two right lines thro' the poles of its ſides; the intercepted arch of the primitive, will be equal to that angle.*

For the diſtance of the poles is equal to that angle.

PROP. X.

25. *If* QH, NK *be two equal circles, whereof* NK
26. *is as far from the projecting point as* QH *from its pole* P; *and if they be projected into the circles whoſe radii are* MC *or* CL, *and* DF *or* FG, F *being the center of* DG, *and* E *the projected pole. I ſay, the pole* E *will be diſtant from their centers in proportion to the radii of the circles; that is,* CE : EF : : CL : DF *or* FG.

For ſince NK and ML are parallel, and arch NI $=$ PH, therefore $<$ ELI $=$ NKI (or *n*KI) $=$ GIP; therefore the triangles IEL and IEG are ſimilar, whence EL : EI : : EI : EG. Again the angle EMI $=$ KNI $=$ PIQ, and therefore the triangles IEM and IED are ſimilar, whence EM : EI : : EI : ED. Therefore $EI^2 = EL \times EG =$ EM

EM × ED. Consequently EM : EL : : EG : ED ; Fig.
and by composition $\frac{EM + EL}{2} : \frac{EM - EL}{2} : :$ $\frac{EG + ED}{2} : \frac{EG - ED}{2}$; that is, CM : EC : : FG : FF. *Q. E. D.*

Cor. 1. *Hence if the circle* KN *be as far from* 25.
the projecting point, as QH *is from either of its poles,* 26.
and if E, O, *be its projected poles; then will* EL : EM : : ED : : EG : : OD : OG.

This follows from the foregoing demonstration, and Cor. 4. Prop. VII.

Cor. 2. *Hence also if* F *be the center, and* FD *the* 25.
radius of any circle QH, *and* E, O *the projected* 26.
poles; then EF : DF : : DF : FO.

For it follows from Cor. 1, that $\frac{EG + ED}{2}$: $\frac{EG - ED}{2}$: : OG + OD : $\frac{OG - OD}{2}$.

Cor. 3. *Hence if the circle* DBG, *be as far from* 27.
its projected pole P, as LMN *is from the projecting* 28.
point; and if any right lines be drawn thro' P, *as* MPG, NPK, *they will cut off similar arches* GK, MN *in the two circles.*

For from the centers C, F, draw the lines CN, FK, then since the angles CPN, and FPK are equal, and by this Prop. CP : CN : : FP : FK; therefore (Geom. II. 16.); the triangles PCN and PFK are similar; and the angle PCN $= \angle$ PFK; therefore the arches MN and GK are similar.

Cor. 4. *Hence also if thro' the projected pole* P *of* 27.
any circle DBG, *a right line* BPK *be drawn. Then* 28.
I say the degrees in the arch GK *shall be the measure of* DB *in the projection. And the degrees in* DB, *shall be the measure of* GK *in the projection.*

For

Fig. For (by Prop. IX.) the arch MN is the meaſure of DB, and therefore GK which is ſimilar to MN, will alſo be the meaſure of it.

Cor. 5. *The centers of all projected circles are all beyond the projected poles (in reſpect to the center of the primitives); and none of their centers can fall between them.*

20. Cor. 6. *Hence it follows (by* Cor. 5. *and* Pr. VIII. Cor. 3.*) that all circles that are not parallel to the primitive have equal arches on the ſphere repreſented by unequal arches on the plane of projection.*

For if P be the projected center, then GH is greater than EF.

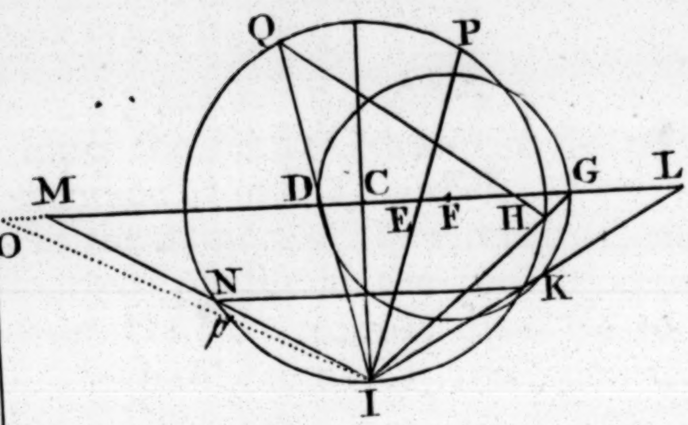
Scholium.

It will be eaſy by the foregoing propoſitions to deſcribe the repreſentation of any circle, and the reverſe will eaſily ſhow what circle of the ſphere any projected circle repreſents. What follows hereafter is deduced from the foregoing propoſitions, and will eaſily be underſtood without any other demonſtration.

If the ſphere was to be projected on any plane parallel to the primitive, 'tis all the ſame thing. For the cones of rays iſſuing from the projecting point, are all cut by parallel planes into ſimilar ſections, it only makes the projections bigger or leſs, according to the diſtance of the plane of projection, whilſt they are ſtill ſimilar; and amounts to no more than projecting from different ſcales upon the ſame plane. And therefore the projecting the ſphere on the plane of a leſſer circle is only projecting it upon the great circle parallel thereto, and continuing all the lines of the ſcheme to that leſſer circle.

PROP.

25.



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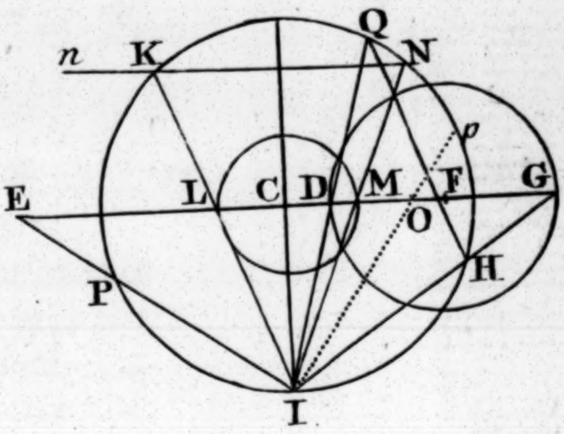

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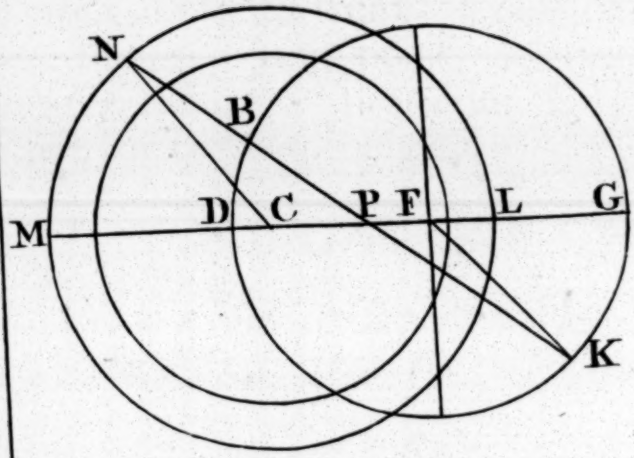

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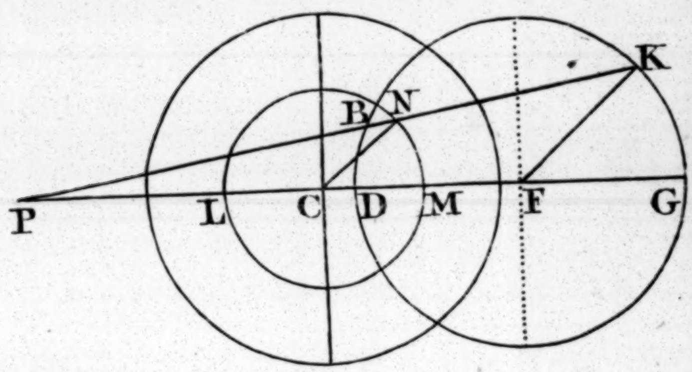

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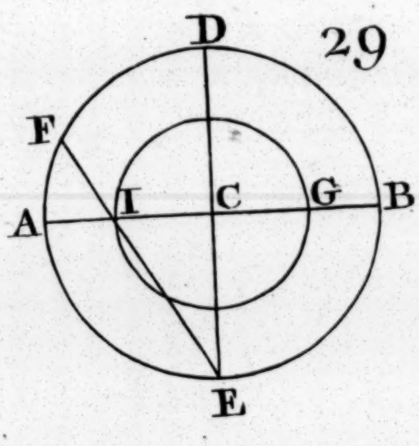

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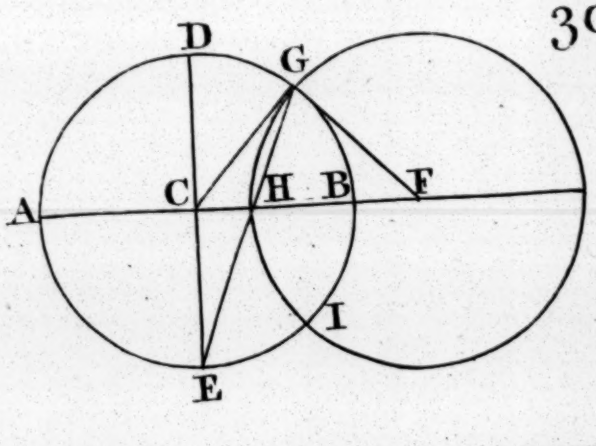

Fig.

PROP. XI. *Prob.*

To draw a circle parallel to the primitive at a given distance from its pole.

Rule.

Thro' the center C draw two diameters AB, DE, perpendicular to one another. Take in your compasses the distance of the circle from the pole of the primitive opposite to the projecting point, and set it from D to F; from E draw EF to intersect AB in I; with the radius CI, and center C, describe the circle GI required. 29.

By the plain Scale.

With the radius CI, equal to the semi-tangent of the circles distance from the pole of projection opposite the projecting point, describe the circle IG. Here the radius of projection CA, is the tangent of 45°, or the sem-itangent of 90°.

PROP. XII. *Prob.*

To draw a lesser circle perpendicular to the primitive at a given distance from the pole of that circle.

Rule.

Thro' the pole B draw the line of measures AB, make BG the circle's distance from its pole, and draw CG, and GF perpendicular to it; with the radius FG describe the circle GI required. 30.

By the Scale.

Set the secant of the circle's distance from its pole from C to F, gives the center. With the tangent of that distance for a radius, describe the circle GI.

Or thus, make BG the circle's distance from its pole; and GF its tangent, set from G, gives F the center;

Fig. center; thro' G deſcribe the circle GI from the cen-
30. ter F.

Cor. *Hence a great circle perpendicular to the primitive, is a right line* CDE *drawn thro' the center perpendicular to the line of meaſures.*

SCHOLIUM.

When the center F lyes at too a great a diſtance; draw EG, to cut AB in H; or lay the ſemi-tangent of DG from C to H. And thro' the three points G, H, I, draw a circle with a bow.

PROP. XIII. *Prob.*

To deſcribe an oblique circle at a given diſtance from a pole given.

Rule.

31. Draw the line of meaſures AB thro' the given point *p*, if that point is given; and draw DE ⊥ to it, alſo draw E*p*P. Or if the point *p* is not given, ſet the height of the pole above the primitive from B to P. Then from P ſet of PH = PI = circle's diſtance from its pole; and draw EH, EI, to interſect AB in F and G. About the diameter FG deſcribe the circle required.

By the Scale.

If the point P is given, apply C*p* to the ſemi-tangents and it gives the diſtance of the pole from D, the pole of projection oppoſite to the projecting point. This diſtance being had, you'll eaſily find the greateſt and neareſt diſtances of the circle from the pole of the primitive oppoſite to the projecting point; take the ſemi-tangents of theſe diſtances and ſet from C to G and F, both the ſame way if the circle lye all on one ſide, but each its own way, if on different ſides of D. And then FG is the diameter of the circle required to be drawn.

Cor.

Cor. 1. *If* F *be the pole of a great circle as of* DLE. *Draw* EFH, *and make* HP = DH, *and draw* EpP, *and then P is its center.* Fig. 31.

Or thus, draw EFH *thro' the pole* F, *make* HK 90 *degrees; draw* EK *cutting the line of measures in* L. *Thro' the three points* D, L, E, *draw the great circle required.*

Cor. 2. *Hence it will be easy to draw one circle parallel to another.*

PROP. XIV. *Prob.*

Thro' two given points A, B, *to draw a great circle.*

Rule.

Thro' one of the points A, draw a line thro' the center, ACG; and EF perpendicular to it. Then draw AE, and EG perpendicular to it. Thro' the three points A, B, G draw the circle required. 32.

Or thus; From E (found as before) draw EH, and then HCI, and lastly EIG, gives G a third point, thro' which the circle must pass.

By the Scale.

Draw ACG; and apply AC to the semi-tangents, find the degrees, set the semi-tangent of its supplement from C to G, for a third point.

Or thus; Apply AC to the tangents, and set the tangent of its complement from C to G. And thro' the three points ABG, describe the circle required.

For since HEI or AEG is a right angle, therefore A, G are opposite points of the sphere; and therefore all circles passing thro' A and G are great circles.

Scholium.

If the points A, B, G lie nearly in a right line, then you may draw a circle thro' them with a bow.

PROP.

Fig.

PROP. XV. *Prob.*

About a pole given, to deſcribe a circle thro' a given point.

Rule.

33. Let P be the pole, and B the given point; thro' P, B deſcribe the great circle AD (by Prop. XIV.), whoſe center is E; thro' the center C draw CPH; and from the center E, draw EB, and BF perpendicular to it. To the center F, and radius FB deſcribe the circle BGH required.

PROP. XVI. *Prob.*

To find the poles of any circle FNG.

Rule.

31. Thro' its center draw the line of meaſures AG, and DE perpendicular to it. Draw EFH, and ſet its diſtance (from its own pole) from H to P, and draw E*p*P, then *p* is the pole.

Or thus, Draw EFH, EIG, and biſect HI in P, and draw E*p*P, and *p* is the internal pole. Laſtly draw PCQ, and EQ*q*, and *q* is the external pole.

In a great circle DLE, draw ELK, and make DH = AK, (or KH = AD, and draw EFH, and F is the pole.

By the Scale.

Apply CF to the ſemi-tangents, and note the degrees. Take the ſum of theſe degrees and of the circle's diſtance from its pole, if the circle lie all on one ſide, but their difference if it encompaſſes the pole of projection; ſet the ſemi-tangent of this ſum or difference from C to the internal pole *p*. And the ſemi-tangent of its ſupplement C*q*, gives the external pole *q*.

Or thus, Apply CF and CG to the ſemi-tangents, ſet the ſemi-tangent of half the ſum of the degrees

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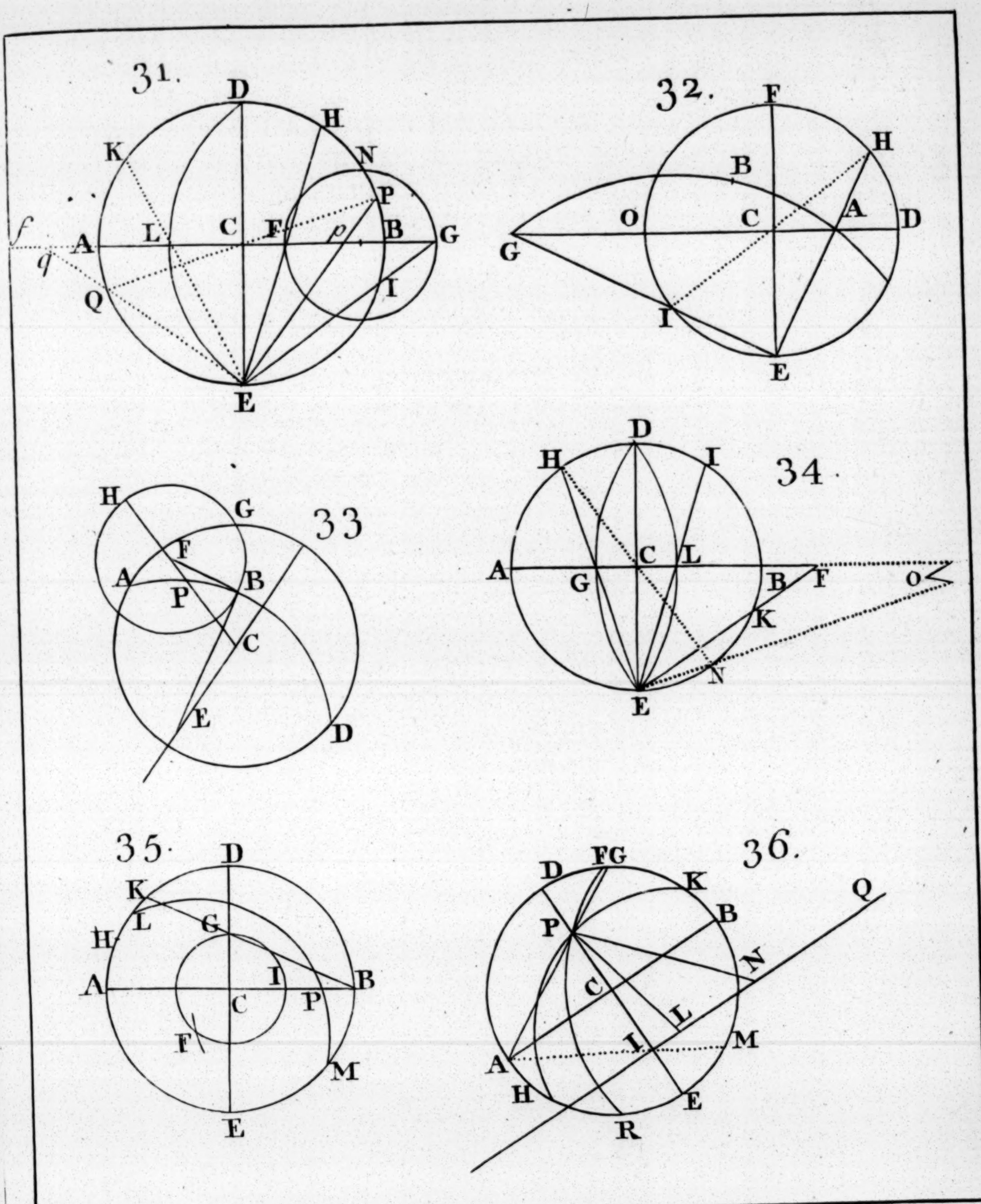

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(if the circle lies all one way) or of half the difference (if it encompasses the pole of projection), from C to the pole p; and the semi-tangent of the supplement, Cq gives the external pole q. Fig. 31.

In a great circle as DLE, draw the line of measures AB perp. to DE; and set the tangent and co-tangent of half its inclination, from the center C, different ways to F and f; which gives the internal and external poles F and f.

PROP. XVII. *Prob.*

To draw a great circle at any given inclination above the primitive; or making any given angle with it, at a given point.

Rule.

Draw the line of measures AB; and DCE perpendicular to it. Make EK = 2HD = twice the complement of the circle's inclination; (or DK = 2AH = twice the inclination); and draw EKF, then F is the center of EGD, the circle required. 34.

Or thus; Draw DE and AB perp. to it, and let D be the point given. Make AH the inclination, and draw EGH and HCN; and ENO, to cut AB in O. Then bisect GO in F, for the center of the circle required.

By the Scale.

Set the tangent of the inclination in the line of measures from C to F, then F is the center. Set the semi-tangent of the complement from C to G; then GF or DF is the radius.

Or the secant of the inclination set from G or D to F gives the center.

Cor. *To draw an oblique circle to make a given angle with a given oblique circle* DGE *at* D. *Draw* EGH, *and set the given angle from* H *to* I, *and draw* ELI. *Thro'* D, L, E *describe a great circle.*

PROP.

Fig.

PROP. XVIII. *Prob.*

Through a given point P, *to draw a great circle, to make a given angle with the primitive.*

Rule.

35. Thro' the point given P and the center C draw the line AB; and DE perpendicular to it. Set the given angle from A to H and from H to K, and draw BGK; with radius CG, and center C describe the circle GIF; and with radius BG and center P cross that circle in F. Then with radius FP and center F, describe the circle LPM required.

By the Scale.

With the tangent of the given angle and one foot in C, describe the arch FG. With the secant of the given angle and one foot in the given point P, cross that arch at F. From the center F describe a circle thro' the point P.

PROP. XIX. *Prob.*

To draw a great circle to make a given angle with a given oblique circle FPR, *at a given point* P, *in that circle.*

Rule.

36. Thro' the center C and the given point P, draw the right line DE; and AB perpendicular to it; draw APG and make BM = 2DG; and draw AM to cut DE in I. Draw IQ perpendicular to DE, then IQ is the line wherein the centers of all circles are found which pass thro' the point P. Find N the center of the given circle FPR, and make the angle NPL equal to the given angle, then L is the center of the circle HPK required.

By

By the Scale. Fig.

Thro' P and C draw DE; apply CP to the semi-tangents, and set the tangent of its complement from C to I (or the secant from P to I). On DI erect the perpendicular IQ. Find the center N of FPR, and make the angle NPL = angle given, and L is the center. 36.

Cor. *If one circle is to be drawn perpendicular to another, it must be drawn thro' its poles.*

PROP. XX. *Prob.*

To draw a great circle thro' a given point P, *to make a given angle with a given great circle* DE.

Rule.

About the given point P as a pole (by Prop. 13. Cor. 1.) describe the great circle FG; find I the pole of the given circle DE, and (by Prop. 16.) about the pole I (by Prop. 13.) describe the small circle HKL at a distance equal to the given angle, to intersect FG in H; about the pole H describe (by Prop. 13.) the great circle APB required. 37.

PROP. XXI. *Prob.*

To draw a great circle to cut two given great circles abd, ebf at given angles.

Rule.

Find the poles *s*, *r*, of the two given circles, by Prop. 16. about which draw two parallels *phk*, *pnk*, at the distances respectively equal to the angles given by Prop. 13. the point of intersection P, is the pole of the circle *moq* required. 50.

Cor. *Hence, to draw a right circle to make with an oblique circle, abd, any given angle. Draw a parallel phk at a distance from the pole of the oblique circle, equal to the given angle. Its intersection*

Fig. *f with the primitive, gives the pole of the right circle get required.*

PROP. XXII. *Prob.*

To lay any number of degrees on a great circle, or to measure any arch of it.

Rule.

38. Let AFI be the primitive; find the internal pole P of the given circle DEH (by Prop 16.) lay the degrees on the primitive from A to F, and draw PA, PF, intercepting the part required DE. Or to measure DE, draw PEF and PDA, and AF is its measure, and applied to the line of chords shows how many degrees it is.

Or thus; Find the external pole *p* of the given circle, set the given degrees from I to K, and draw *p*I, *p*K, intercepting the part DE required. Or to measure DE, thro' D and E draw *p*I, *p*K, then KI is the measure of DE.

Or thus; Thro' the internal pole P, draw the lines DPG, and EPL; setting the given degrees from G to L in the circle GL; then DE is the arch required. Or if DE be to be measured, then the degrees in the arch GL is the measure of DE.

Or thus; Set the given degrees from G to H in the circle GL and from the external pole P, draw *p*G, *p*H, intercepting DE the arch required. Or to measure DE, draw *p*DG, *p*EH, then the degrees in GH, is equal to DE.

By the Scale for right Circles.

38. Let CA be the right circle, take the number of degrees off the semi-tangents and set from C to D for the arch CD. Or if the given degrees are to be set from A, then take the degrees off the semi-tangents from 90° towards the beginning, and set from A to D. And if CD was to be measured, apply

apply it to the beginning of the semi-tangents; and Fig. to measure AD, apply it from 90° backwards, and the degrees intercepted gives its measure.

SCHOLIUM.

The primitive is measured by the line of chords, or else it is actually divided into degrees.

PROP. XXIII. *Prob.*

To set any number of degrees on a lesser circle, or to measure any arch of it.

Rule.

Let the lesser circle be DEH; find its internal pole 38. P, by Prop. 16. describe the circle AFK parallel to the primitive, by Prop. 11. and as far from the projecting point, as the given circle DE is from its internal pole P, set the given degrees from A to F, and draw PA, PF intersecting the given circle in D, E; then DE is the arch required. Or to measure DE, draw PDA, PEF, and AF shows the degrees in DE.

Or thus; Find the external pole *p*, of the given circle by Prop. 16. describe the lesser circle AFK as far from the projecting point, as DE the given circle is from its pole *p*, by Prop. 11. set the degrees from I to K and draw *p*DI, *p*EK, then DE represents the given number of degrees. Or to measure DE; draw *p*DI, *p*EK; and KI is the measure of DE.

Or thus; Let O be the center of the given circle DEH; thro' the internal pole P, draw lines DPG, EPL, divide the quadrant GQ into 90 equal degrees, and if the given degrees be set from G to L, then DE will represent these degrees. Or the degrees in GL will measure DE.

Or thus; Divide the quadrant GR into 90 equal parts or degrees, and set the given degrees from G to H, and draw *p*DG, *p*EH, from the external pole *p*; then DE will represent the given degrees. Or

Fig. thro' D, E drawing pDG, pEH, then the number of equal degrees in GH is the measure of DE.

SCHOLIUM.

Any circle parallel to the primitive is divided or measured, by drawing lines from the center, to the like divisions of the primitive. Or by help of the chords on the sector, set to the radius of that circle.

PROP. XXIV. *Prob.*

To measure any angle.

Rule.

By Cor. 1. Prop. 13. About the angular point as a pole, describe a great circle, and note where it intersects the legs of the angle; thro' these points of intersection, and the angular point, draw two right lines, to cut the primitive; the arch of the primitive intercepted between them is the measure of the angle. This needs no example.

Or thus; by Prop. 16. Find the two poles of the containing sides, (the nearest, if it be an acute angle, otherwise the furthest) and thro' the angular point and these poles, draw right lines to the primitive, then the intercepted arch of the primitive is
31. the angle required. As if the angle AEL was required. Let C and F be the poles of EA and EL. From the angular point E, draw ECD and EFH. Then the arch of the primitive DH, is the measure of the angle AEL.

SCHOLIUM.

Because in the Stereographic Projection of the Sphere, all circles are projected either into circles or right lines, which are easily described; therefore this sort of projection is preferred before all others. Also those planes are preferred before others to project upon, where most circles are projected into right lines, they being easier to describe and measure than circles are; such are the projections on the planes of the meridian and solstitial colure.

SECT.

Fig.

SECT. III.

The Gnomonical Projection of the SPHERE.

PROP. I.

Every great circle as BAD *is projected into a right line, perpendicular to the line of measures, and distant from the center, the co-tangent of its inclination; or the tangent of its nearest distance from the pole of projection.* 39.

Let CBED be perpendicular both to the given circle BAD and plane of projection, and then the intersection CF will be the line of measures. Now since the plane of the circle BD, and the plane of projection are both perpendicular to BCDE, therefore their common section will also be perpendicular to BCDE, and consequently to the line of measures CF. Now since the projecting point A is in the plane of the circle, all the points of it will be projected into that section; that is, into a right line passing thro' *d*, and perpendicular to C*d*. And C*d* is the tangent of CD, or co-tangent of C*d*A. *Q. E. D*

Cor. 1. *A great circle perpendicular to the plane of projection is projected into a right line passing thro' the center of projection; and any arch is projected into its correspondent tangent.* 39.

Thus the arch CD is projected into the tangent C*d*.

Cor. 2. *Any point as* D, *or the pole of any circle, is projected into a point d distant from the pole of projection* C, *the tangent of that distance.*

Cor. 3. *If two great circles be perpendicular to each other, and one of them passes thro' the pole of projection*

Fig. *tion; they will be projected into two right lines per-*
39. *pendicular to each other.*

For the reprefentation of that circle which paffes thro' the pole of projection is the line of meafures of the other circle.

Cor. 4. *And hence if a great circle be perpendicular to feveral other great circles, and its reprefentation pafs thro' the center of projection; then all thefe circles will be reprefented by lines parallel to one another, and perpendicular to the line of meafures or reprefentation of that firft circle.*

PROP. II.

39. *If two great circles interfect in the pole of projection; their reprefentations fhall make an angle at the center of the plane of projection equal to the angle made by thefe circles on the fphere.*

For fince both thefe circles are perpendicular to the plane of projection; the angle made by their interfections with this plane, is the fame as the angle made by thefe circles. *Q. E. D.*

PROP. III.

Any leffer circle parallel to the plane of projection is projected into a circle, whofe center is the pole of projection; and radius the tangent of the circle's diftance from the pole of projection.

39. Let the circle PI be parallel to the plane GF, then the equal arches PC, CI are projected into the equal tangents GC, CH; and therefore C the point of contact and pole of the circle PI and of the projection, is the center of the reprefentation GH. *Q. E. D.*

Cor. *If a circle be parallel to the plane of projection, and 45 degrees from the pole, it is projected into a circle*

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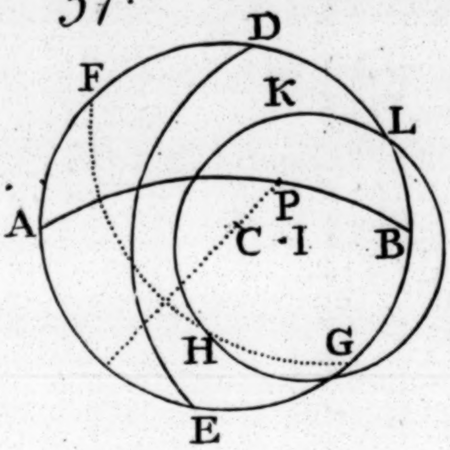
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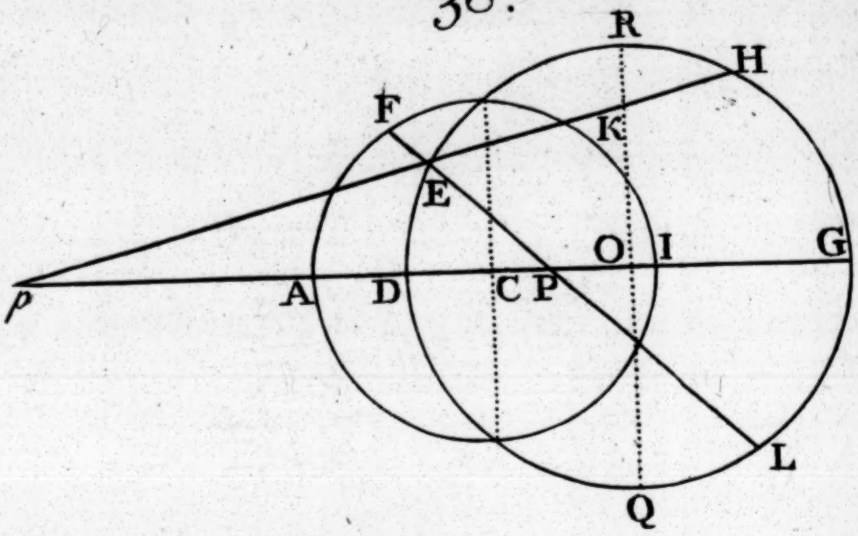
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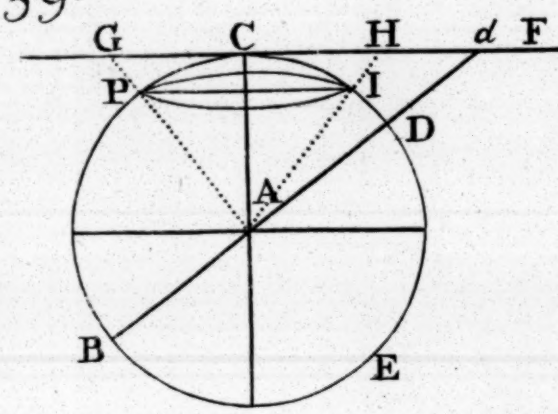
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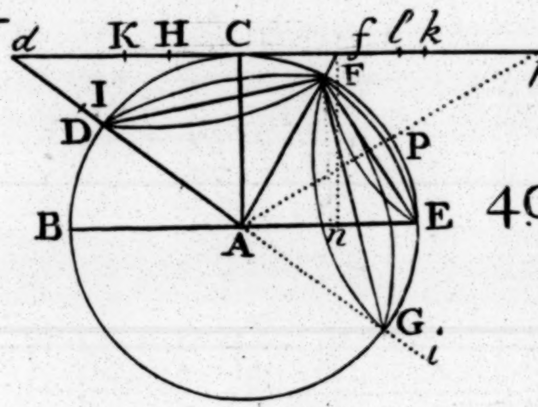
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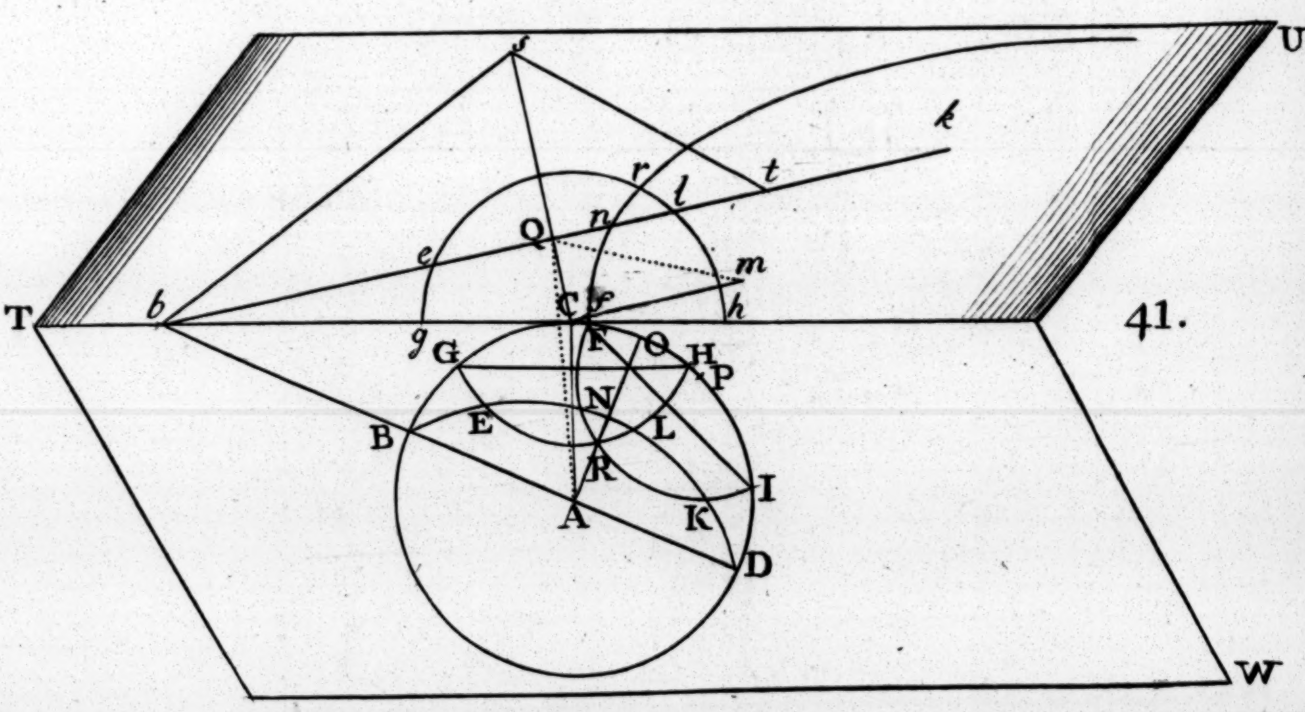
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41.

a circle equal to a great circle of the sphere; and may Fig.
therefore be looked upon as the primitive circle in this projection, and its radius the radius of projection.

PROP. IV.

Every lesser circle (not parallel to the plane of projection) is projected into a conic section, whose transverse axis is in the line of measures, and whose nearest vertex is distant from the center of the plane the tangent of its nearest distance from the pole of projection; and the other vertex is distant the tangent of its furthest distance. 40.

Let BE be parallel to the line of measures *dp*, then any circle is the base of a cone whose vertex is at A, and therefore that cone being produced will be cut by the plane of projection in some conic section; thus the circle whose diameter is DF will be cut by the plane in an ellipsis whose transverse is *df*; and C*d* is the tangent of CAD, and C*f* of CF. In like manner the cone AFE being cut by the plane, *f* will be the nearest vertex; and the other point into which E is projected is at an infinite distance. Also the cone AFG (whose base is the circle FG) being cut by the plane *f* is the nearest vertex; and GA being produced gives *d* the other vertex. *Q. E. D.*

Cor. 1. *If the distance of the furthest point of the circle be less than 90° from the pole of projection, then it will be projected into an ellipsis.*

Thus DF is projected into *df*, and DC being less than 90°, the section *df* is an ellipsis, whose vertices are at *d* and *f*; for the plane *df* cuts both sides of the cone, *d*A, *f*A.

Cor. 2. *If the furthest point be more than 90 degrees from the pole of projection, it will be projected*

 into

Fig. *into an hyperbola. Thus the circle* FG *is projected into*
40. *an hyperbola whose vertices are f and d, and transverse fd.*

For the plane *dp* cuts only the side A*f* of the cone.

Cor. 3. *And in the circle* EF, *where the furthest point* E *is* 90° *from* C; *it will be projected into a parabola, whose vertex is f.*

For the plane *dp* (cutting the cone FAE) is parallel to the side AE.

Cor 4. *If H be the center, and* K, *k*, *l*, *the focus of the ellipsis, hyperbola, or parabola*; *then* HK $= \frac{Ad - Af}{2}$ *for the ellipsis, and* H*k* $= \frac{Ad + Af}{2}$ *for the hyperbola*; *and (drawing fn perpendicular on* AE) $fl = \frac{nE + Ff}{2}$, *for the parabola*; *which are the representations of the circles* DF, FG, FE *respectively.*

This all appears from the Conic Sections.

PROP. V.

41. *Let the plane* TW *be perpendicular to the plane of projection* TV, *and* BCD *a great circle of the sphere in the plane* TW. *And let the great circle* BED *be projected in the right line bek. Draw* CQS $\perp$ *bk, and* C*m* $\parallel$ *to it and equal to* CA, *and make* QS $=$ Q*m*; *then I say any angle* Q*st* $=$ Q*t*.

Suppose the hypothenuse AQ to be drawn, then since the plane ACQ is perpendicular to the plane T*v*, and *b*Q is $\perp$ to the intersection CQ, therefore *b*Q is perpendicular to the plane ACQ, and consequently *b*Q is perpendicular to the hypothenuse AQ. But AQ $=$ Q*m* $=$ Q*s*, and Q*s* is also perpendicular to *b*Q. Therefore all angles made at S cut the line *b*Q in the same points as the angles made

at

at A; but by the angles at A the circle BED is projected into the line *b*Q. Therefore the angles at *s* are the measures of the parts of the projected circle *b*Q; and *s* is the dividing center thereof. *Q. E. D.* Fig. 41.

Cor. 1. *Any great circle tQb is projected into a line of tangents to the radius* SQ.

For Q*t* is the tangent of the angle QS*t* to the radius QS or Q*m*.

Cor. 2. *If the circle b*C *pass thro' the center of projection; then* A *the projecting point is the dividing center thereof. And* C*b is the tangent of its correspondent arch* CB, *to* CA *the radius of projection.*

PROP. VI.

Let the parallel circle GEH *be as far from the pole of projection* C *as the circle* FKI *is from its pole* P; *and let the distance of the poles* C, P *be bisected by the radius* AO, *and draw b*AD *perpendicular to* AO; *then any right line bek drawn thro' b, will cut off the arches bl* = F*n*, *and ge* = *kf (supposing f the other vertex), in the representations of these equal circles in the plane of projection.* 41.

For let G, E, R, L, H, N, R, K, I be respectively projected into the points *g*, *e*, *r*, *l*, *h*, *n*, *r*, *k*, *f*. Then since in the sphere, the arch BF = DH, and arch BG = DI. And the great circle BEKD makes the angles at B and D equal, and is projected into a right line as *bl*; therefore the triangular figures BFN and DHL are similar, and equal; and likewise BGE, and DIK are similar and equal, and LH = NF, and KI = EG; whence it is evident their projections *lh* = *n*F, and *kf* = *ge*. *Q. E. D.*

PROP.

Fig.

PROP. VII.

42. *If hlg and* F*nk be the projections of two equal circles, whereof one is as far from its pole* P *as the other from its pole* C; *which is the center of projection; and if the distance of the projected poles* C, *p be divided in o, so that the degrees in* C*o*, *op*, *be equal, and the perpendicular o*S *be erected to the line of measures gh. I say the lines pn,* C*l*, *drawn from the poles* C, *p thro' any point* Q *in the line o*S, *will cut off the arch* F*n* $= \angle$ QC*p*.

For drawing the great circle GPI, in a plane perpendicular to the plane of projection. The great circle AO perpendicular to CP is projected into *o*S by Prop. I. Cor. 3. Now let Q be the projection of *q*, and since *p*Q, CQ are right lines, therefore they represent the great circles P*q*, C*q*. But the spherical triangle P*q*C is an isoceles-triangle, and therefore the angles at P and C are equal. But because P is the pole of FI, therefore the great circle P*q* continued, will cut an arch off FI $= \angle$ CP*q* $= \angle$ PC*q* $= \angle$ QC*p* by Prop. II. That is (since F*n* represents the part cut off from FI) arch F*n* = arch *lh* or $\angle$ QC*h*. *Q. E. D.*

Cor. *Hence if from the projected pole p of any circle, a perpendicular be erected to the line of measures; it will cut off a quadrant from the representation of that circle.*

For that perpendicular will be parallel to OS; Q being at an infinite distance.

PROP.

Fig.

PROP. VIII.

Let F*nk be the projection of any circle* FI, *and p* 42.
the projected pole P. *And if* C*g be the co-tangent of*
CAP, *and g*B *perpendicular to the line of measures*
*g*C, *and* CAP *be bisected by* AO, *and the line o*B, *be*
drawn to any point B, *and also p*B *cutting* F*nk in d.*
*I say the angle go*B $=$ *arch* F*d*.

For the arch PG is a quadrant, and the $\angle goA = \angle gpA + \angle oAp =$ (because GCA and gAp are right angles) $gAC + oAp = gAC + CAo = \angle gAo$. Therefore $gA = go$, consequently o is the dividing center of gB the representation of GA; and consequently by Prop. V. $\angle go$B is the measure of gB. But since pq represents a quadrant, therefore p is the pole of gB, and therefore the great circle pdB passing thro' the pole of the circles gB and Fn will cut off equal arches in both, that is F$d = g$B $= \angle go$B. *Q. E. D.*

Cor. *The* $\angle go$B *is the measure of the angle gp*B.
For the triangle gpB represents a triangle on the sphere wherein the arch which gB represents is equal to the angle which $\angle p$ represents, because gp is 90 degrees. Therefore goB is the measure of both.

Scholium.

Thus far I have treated of the theory; what follows is the practical part, and depends altogether on what is above delivered, in which I think no difficulty can occur. In the Gnomonical Projection, the plane projected on, is supposed to touch the hemisphere to be projected, in its vertex; and the point of contact will be *the center of projection*. But if it be required to project upon any plane parallel

Fig. rallel to this touching plane, the procefs will be no
42. way different, and is only taking a greater or leffer radius of projection, according to the greater or leffer diftance; which is in effect projecting a greater or leffer fphere upon its touching plane.

When you have the fphere to project this way, upon a given plane; it will affift the imagination, if you fuppofe yourfelf placed in the center of the fphere with your face towards the plane, whofe pofition is given; and from thence projecting with your eye, the circles of the fphere upon this plane.

PROP. IX. *Prob.*

43. *To draw a great circle, thro' a given point, and at a given diftance from the pole of projection.*

Rule.

Defcribe the circle ADB with the radius of projection, and thro' the given point P draw the right line PCA, and CE perpendicular to it; make the angle CAE = given diftance of the circle from C, and thro' E defcribe the circle EFG, and thro' P draw the line PK touching the circle in I, then is PIK the circle required.

By the plain Scale.

With the tangent of the circle's diftance from the pole of projection C, defcribe the circle EIF, and draw PK to touch this circle; and PIK is the circle required.

PROP. X. *Prob.*

43. *To draw a great circle perpendicular to a given great circle, which paffes thro' the pole of projection; and at a given diftance from that pole.*

Rule.

Draw the primitive ADB. Let CI be the given circle, draw CL perpendicular to CI, and make the angle

angle CLI = the given distance; thro' I draw KP Fig.
parallel to CL for the circle required. 43.

By the Scale.

In the given circle CI, set the tangent of the given distance, from C to I; thro' I draw KP perpendicular to CI, then KP is the circle required.

PROP. XI. *Prob.*

To measure any part of a great circle; or to set any number of degrees thereon. 44.

Rule.

Let EP be the great circle; thro' C draw ID perpendicular to EP, and CB parallel to it. Let EBD be a circle described with the radius of projection CB, make IA = IB; then A is the dividing center of EP, consequently drawing AP, the ∢ IAP = measure of the given arch IP.

Or if the degrees be given, make the ∢ IAP = these given degrees, which cuts off IP, the arch correspondent thereto.

By the Scale.

Draw ICD perpendicular to EP; apply CI to the tangents, and set the semi-tangent of its complement from C to A, gives the dividing center of EP, &c.

PROP. XII. *Prob.*

To draw a great circle to make a given angle with a given great circle, at a given point; or to measure an angle made by two great circles. 51.

Rule.

Let P be the given point, and PB the given great circle. Draw thro' P, and C the center of projection, the line PCG, to which from C draw CA perpendicular,

Fig. 51. dicular, and equal to the radius of projection. Draw PA and AG perpendicular to it, at G erect BD perpendicular to GC, cutting PB in B; draw AO bisecting the angle CAP; then at the point O, make BOD = angle given, and from D draw the line DP, then BPD is the angle required.

Or if the degrees in the angle BPD be required, from the points B, D, draw the lines BO, DO; and the angle BOD is the measure of BPD.

Cor. *If an angle be required to be made at the pole or center of projection, equal to a given angle; this is no more than drawing two lines from the center making the angle required. And if one great circle be to be drawn* ⊥ *to another great circle, it must be drawn thro' its pole.*

PROP. XIII. *Prob.*

43. *To project a lesser circle parallel to the primitive.*

Rule.

With the radius of projection AC, and center C, describe the primitive circle ADB, by Cor. Prop. III. and draw ACB, and GCE perpendicular to it.

Set the circle's distance from its pole from B to H, and from H to D, and draw AED. With radius CE describe the circle EFG required.

By the Scale.

With the radius CE equal to the tangent of the circle's distance from its pole, describe the circle EFG, for the circle required.

PROP. XIV. *Prob.*

48. *To draw a lesser circle perpendicular to the plane of projection.*

Rule.

Thro' the center of projection C, draw its parallel great circle TI. At C make the angle ICN and

and TCO = the given circle's distance from its parallel great circle TI; make CL equal radius of projection, and draw LM perpendicular to CL. Set LM from C to V, or CM from C to F. Then thro' the vertex V between the assymptotes CN, CO describe the hyperbola WVK. Or to the focus F, and semi-transverse CV, describe the hyperbola; for the circle required. Fig. 48.

Otherwise by Points.

Thro' the center of projection C draw the line of measures CF, and TCI perpendicular to it, draw any number of right lines CV, DE, GH, IK &c. and PQ, RS, TW, &c. perpendicular to TI. And by Prop. XI. make CV, DE, GH, &c. each equal to the distance of the given circle from its parallel great circle; then all the points W, S, Q, V, E, H, K, &c. joined by a regular curve will be the representation of the circle required.

Or thus.

Make the angle *iak* = distance of the given circle from its parallel great circle. Then thro' the center of projection C, draw the great circle TCI parallel to the circle given, upon which erect the perpendicular CA = radius of projection. Also draw any number of right lines CV, DE, GH, IK, &c. perpendicular to TI. Then take each of the distances from A to C, D, G, I, &c. and set them from *a* to *c*, *g*, *d*, *i*, &c. and to *ai* draw the perpendiculars *cv*, *de*, *gh*, *ik*, &c. and make CV, DE, GH, IK, &c. respectively equal to *cv*, *de*, *gh*, *ik*, &c. which gives the points V, E, H, K, &c. after the same manner on the other side, find the points Q, S, W, &c. then thro' all these points W, S, Q, V, E, H, K, &c. draw a regular curve, which will be an hyperbola representing the circle given.

By

Fig. *By the Scale.*

48\. Take the tangent of the circle's diſtance from its parallel great circle, and ſet it from C (the center of projection) to V, and the ſecant thereof from C to F. Then with the ſemi-tranſverſe CV, and focus F, deſcribe the hyperbola WVHK.

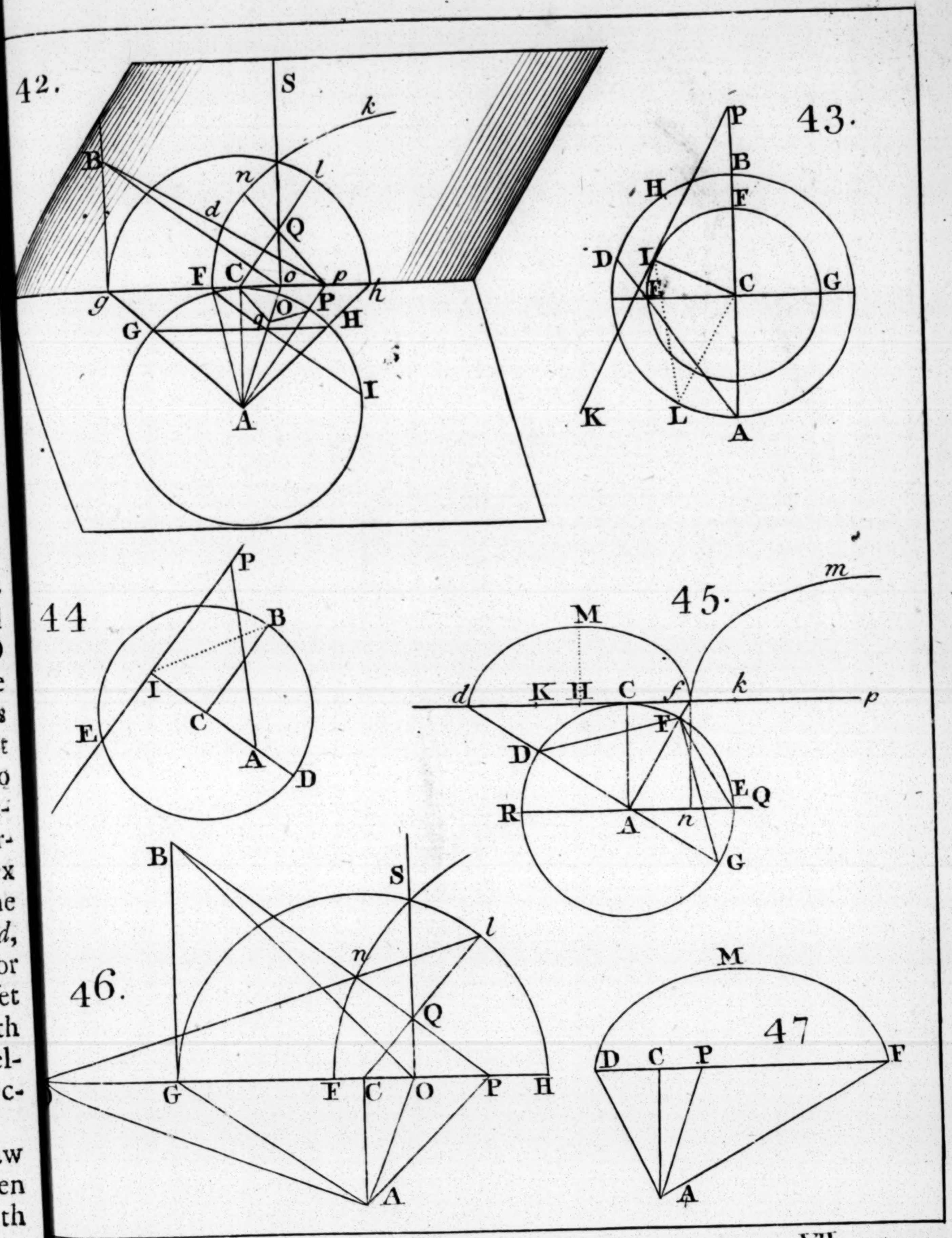
PROP. XV. *Prob.*

To project any leſſer oblique circle given.

Rule.

Draw the line of meaſures *dp*, and at C the
45\. center of projection draw CA ⊥ to *dp* and = radius of projection; with the center A, deſcribe the circle DCFG; and draw RAE parallel to *dp*. Then take the greateſt and leaſt diſtances of the circle from the pole of projection and ſet from C, to D and F, for the circle DF; and from A, the projecting point, draw AF*f*, and AD*d*, then *df* will be the tranſverſe axis of the ellipſis. But if D fall beyond the line RE, as at G, then draw a line from G backward thro' A to D, and then *df* is the tranſverſe of an hyperbola. But if the point D fall in the line RE as at E, then the line AE no where meets the line of meaſures, and the projection of E is at an infinite diſtance, and then the circle will be projected into a parabola whoſe vertex is *f*. Laſtly, biſect *df* in H the center, and for the ellipſis take half the difference of the lines A*d*, A*f*, and ſet from H to K for the focus. But for the hyperbola take half the ſum of A*d*, A*f*, and ſet from H to the focus *k* of the hyperbola. Then with the tranſverſe *df* and focus K or *k* deſcribe the ellipſis *d*M*f*, or the hyperbola *fm*. For the projection of the circle given.

But for the parabola make EQ = F*f*, and draw *fn* ⊥ AQ, and ſet $\frac{1}{2}n$Q from *f* to K the focus. Then with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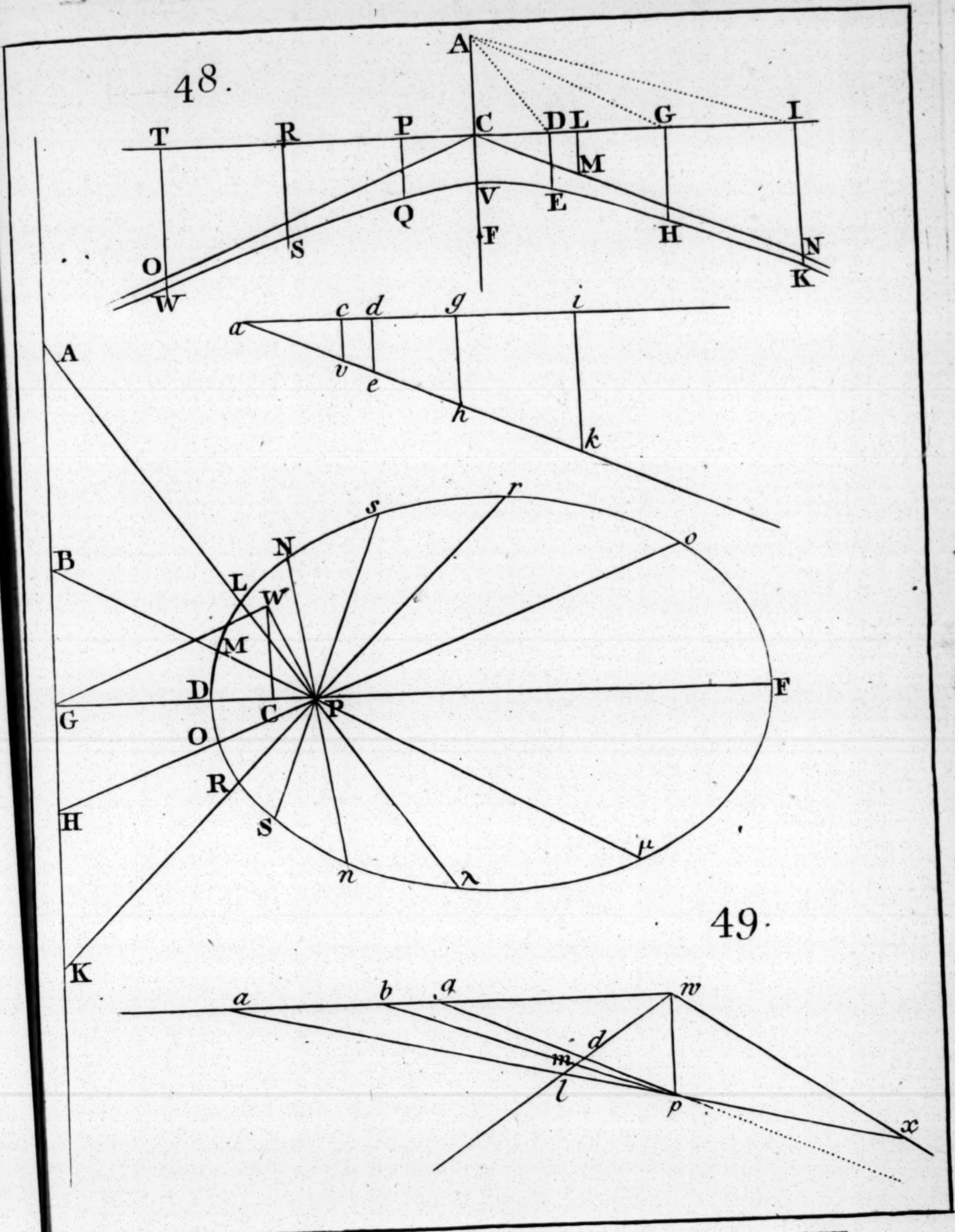

Projection.

.VIII. p. 44.

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with the vertex *f* and focus *k* describe the parabola *fm*, for the projection of the given circle FE. Fig.

Otherwise by Points.

Thro' the center of projection C, draw the line of measures CF, passing thro' the pole P (if P is given; but if not, find it, by setting off CP = the distance of that pole, from the center of projection, by Prop. XI.) then set off PD, PF equal to the given distance from its pole, by Prop. XI. Thro' P draw a sufficient number of right lines, Lλ, Mμ, N*n*, O*o*, R*r*, S*s*, &c. which will all represent great circles. Find the dividing centers of each of these lines; and by Prop. XI. set off upon each of them from P, the given distance of the circle from its pole, as PL, Pλ, PM, Pμ, &c. and thro' all the points L, M, D, O, R, &c. draw a curve line, for the circle required. 49.

Or thus.

Draw the line of measures PCG, and by Prop. XI. make CG = the distance of the parallel great circle from the pole of projection, and draw AGK perpendicular to it, which will represent a great circle whose pole is P. Draw any number of right lines thro' P to AK, as AP, BP, HP, &c. and by Prop. XI. set off from AK the parts AL, BM, HO, &c. each equal to the circle's distance from its parallel great circle. Then all the points L, M, D, O, &c. being joined by a regular curve, will represent the parallel circle required. 49.

Or thus.

Thro' the center of projection C draw the line of measures DCF, and the radius of projection CW perpendicular to it, and AGK ⊥ GC, for a great circle whose pole is P. Draw *wp* = WP, and *wa* ⊥ to it, draw any number of right lines, AP, BP, GP, &c. and make *pg*, *pb*, *pa*, &c. = PG, PB, PA, 49.

Fig. &c. alſo make the $<$ *pwl* and *pwx* $=$ the circle's
49. diſtance from its pole P (or *awl* $=$ the diſtance from its parallel great circle); and upon PG, PB, PA, &c. make PD, PM, PL, &c. $=$ *pd*, *pm*, *pl*, &c. reſpectively.

Or make GD, BM, AL, &c. $=$ *gd*, *bm*, *al*, &c. After the ſame manner, find the points O, R, &c. and thro' all the points R, O, D, M, L, &c. draw a regular curve, making no angles, which will repreſent the parallel required. Likewiſe where any line *ap* cuts *wx*, that diſtance from *p* will give the point λ, or is $=$ Pλ; and ſo of any other of the lines *bp*, *gp*, &c.

The reaſon of this proceſs will be plain, if you ſuppoſe the points p, w applied to P, W; *and g, b, a, &c. ſucceſſively to* G, B, A, &c. *for then d, m, l, will fall upon* D, M, L, &c.

By the Scale.

45. Take the tangents of the circle's neareſt and furtheſt diſtance from the pole of projection, and ſet from C to *f* and *d*, gives the vertices, and biſect *df* in H; then take half the difference, or half the ſum, of the ſecants of the greateſt and leaſt diſtances from the pole of projection, and ſet from H, to K or *k* for the focus of the ellipſis or hyperbola, which may then be deſcribed.

49. Cor. *If the curve be required to paſs thro' a given point* S; *meaſure* PS *by Prop.* XI, *and then the curve may be drawn by this Problem.*

PROP. XVI. *Prob.*

47. *To find the pole of any circle in the projection*, DMF.

Rule.

From the center of projection C, draw the radius of projection CA perpendicular to the line of meaſures

sures DF. And to A the projecting point, draw Fig.
DA, FA, and bisect the angle DAF by the line AP, 47.
then P is the pole. But if the curve be an hyperbola, as *fm*, *fig.* 45, you must produce *d*A, and bisect the angle *f*AG. And in a parabola, where the point *d* is at an infinite distance, bisect the angle *f*AE.

Or thus; Drawing CA perpendicular to DC, draw DA, and make the angle DAP = the circle's distance from its pole, gives the pole P.

By the Scale.

Draw the radius of projection CA ⊥ to the line of measures DF. Apply CD CF to the tangents, and set the tangent of half the difference of their degrees from C to P, if D, F lye on contrary sides of C; but half the sum if on the same side, gives P the pole.

Or thus; By Prop. XI. set off from D to P, the circle's distance from its pole, gives the pole P.

Cor. *If it be a great circle as* BG; *draw the line of measures* GC, *and* CA ⊥ *to it, and equal to the radius of projection*; *make* GAP *a right angle, and* P *is the pole.* 46.

PROP. XVII. *Prob.*

To measure any arch of a lesser circle; or to set any number of degrees thereon.

Rule.

Let F*n* be the given circle. From the center of projection C, draw CA perpendicular to the line of measures GH. To P the pole of the given circle draw AP, and AO bisecting the angle CAP. And draw AD perpendicular to AO. Describe the circle G*l*H (by Prop. XIII.) as far from the pole of projection C, as the given circle is from its pole P. And thro' any given point *n* in the circle F*n*, 48.

Fig. draw D*nl*, gives H*l* the number of degrees = F*n*.
46. Or the degrees being given and ſet from H to *l*, the line D*l* cuts off F*n* equal thereto.

Or thus; AO being drawn as before, erect OS perpendicular to CO; thro' the given point *n* draw P*n* cutting OS in Q, then thro' Q draw C*l*, and the angle QCP is = F*n*. Or making QCP = the degrees given, draw PQ*n*, and arch F*n* = theſe degrees.

Or thus; AO, AP, being drawn as before, draw AG perpendicular to AP, and GB perpendicular to GC. Thro' the given point *n* draw PB cutting GB in B, and draw OB, then the < GOB = arch F*n*. Or making < GOB = the given degrees; draw PB, and it cuts off F*n* = the degrees given.

By the Scale.

Let C be the center of projection, P the pole of the given circle. Apply CP to the tangents, and ſet the tangent of its half from C to O, and the co-tangent of its half from C to D; with radius CG = tangent of the degrees in FP the given circle's diſtance from its pole, deſcribe the circle GSH. Then D*l* drawn thro' *n* or *l*, cuts off H*l* = F*n*.

Or thus; O being found as before, erect OS perpendicular to CO; thro' the given point *n* draw PQ*n*, and < QCH = F*n*.

Or thus; Apply CP to the tangents, and ſet the co-tangent thereof from C to G. Erect GB perpendicular to GC. Thro' *n* draw P*n*B, and draw BO; then < GOB = F*n*.

48. Cor. *If the leſſer circle be perpendicular to the plain of projection as* VHK. *You have no more to do but to draw the perpendiculars* VC, HG, *to its parallel great circle* CI. *Then* CG (*meaſured by Prop*. XI.) *will be equal to* VH; *or the degrees ſet from* C *to* G, *cuts off* VH *equal thereto.*

SCHO-

Fig.

SCHOLIUM.

This sort of projection is little used, by reason of several of the circles of the sphere fall in ellipses and hyperbolas, which are very difficult to describe. Notwithstanding it is very convenient for solving some Problems of the sphere, because all the great circles are projected into right lines. And this sort, or the Gnomonic Projection is the very foundation of all dialling. For if the sphere be projected on any plane, and upon that side of it on which the sun is to shine; and the projected pole be made the center of the dial, and the axis of the globe the Stile or Gnomon, and the radius of projection its height; you will have a dial drawn with all its furniture. Upon this account it deserves to be more taken notice of, than at present it is. I have in the foregoing propositions given, I think, all the fundamental principles of this kind of projection, having met with little or nothing done upon this subject before. 48.

GENERAL PROBLEM.

To project the sphere upon any given plane.

Before you can project the sphere upon any plane, you must have a perfect knowledge of all its circles, and their positions in respect of one another; the distances of the lesser circles from their poles, and from their parallel great circles; the angles made by great circles, or their inclinations, to one another, particularly to the primitive circle, on whose plane (or a parallel thereto) you are about to project the sphere. Then after the primitive circle is described; you must describe all other circles concerned in the Problem, according to the rules of that sort of Projection, you are going to use;

Fig. and the interſection of theſe circles will determine the Problem.

And note, that the Projection of the concave ſide of the ſphere is more fit for aſtronomical purpoſes; for in looking at the heavens, we view the concavity. But it is better to project the convex hemiſphere in geography, becauſe we ſee the convex ſide only.

The principal Points, Angles and Circles of the Sphere are as follows.

I. *Points.*

52. 1. *Zenith* is the point over our heads, Z.

53. 2. *Nadir* is the point under our feet, N.

55. 3. *Poles* of the world are 2 points, round which the diurnal revolution is performed, P the north pole, *p* the ſouth pole. A line drawn through the poles, is called the *Axis* of the world, as P*p*.

4. *The Center* of the earth or of the heavens, C.

5. *Equinoctial Points*, are the points of interſection of the Equator and Ecliptic, ♈, ♎.

6. *Solſtitial Points*, are the beginning of Cancer and Capricorn, ♋, ♑.

II. *Great Circles.*

1. *Equinoctial*, is a circle 90 degrees diſtant from the poles of the world, as EQ.

2. *Meridians*, or *hour Circles*; are circles paſſing thro' the poles of the world, as P ⊙ *p*, PE*p*, &c.

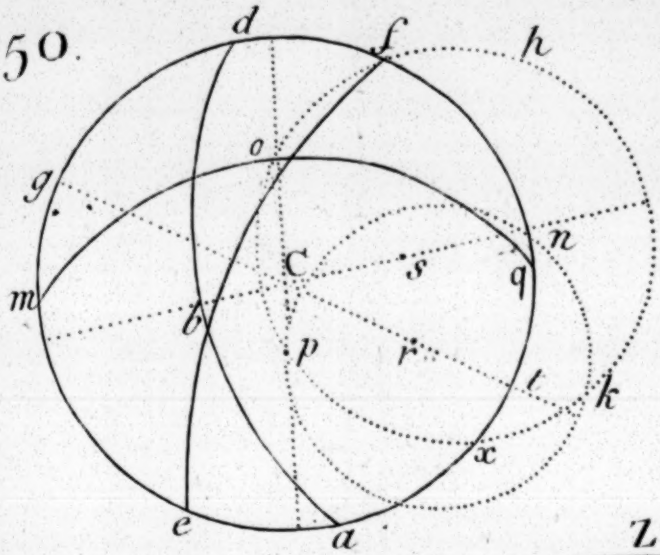
3. *Solſtitial Colure*, is a meridian paſſing thro' the ſolſtitial points, as P♋*p*.

4. *Equinoctial Colure*, is a meridian paſſing thro' the equinoctial points, P C*p*.

5. *Ecliptic* is the circle thro' which the ſun ſeems to move in a year, ♋ ♑; it cuts the equinoctial at an angle of 23° 30′, in paſſing thro' the equinoctial points. In this are reckoned the 12 Sines, ♈, ♉, ♊, ♋, ♌, ♍, ♎, ♏, ♐, ♑, ♒, ♓.

6. *Ho-*

50.



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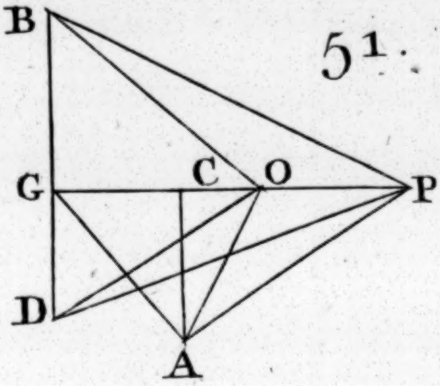

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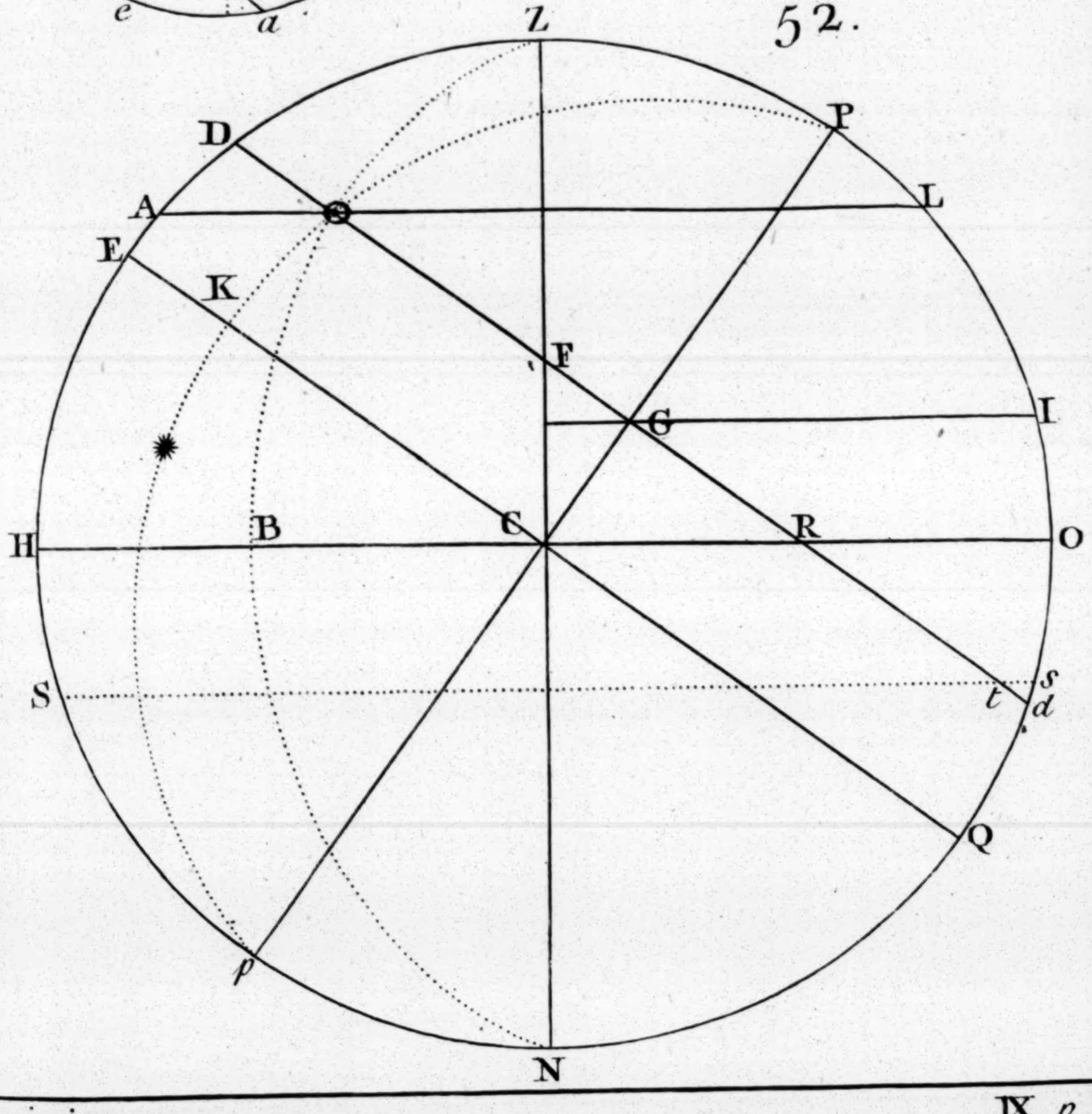

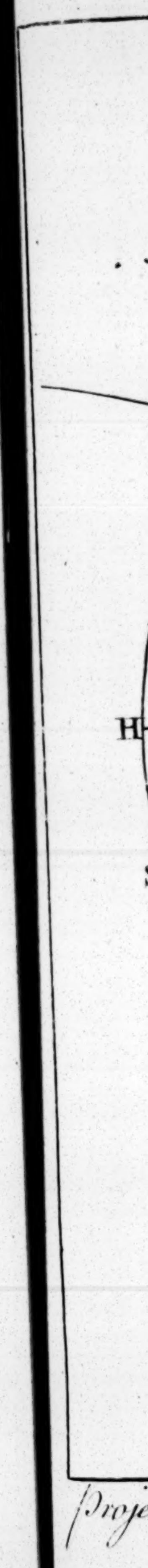

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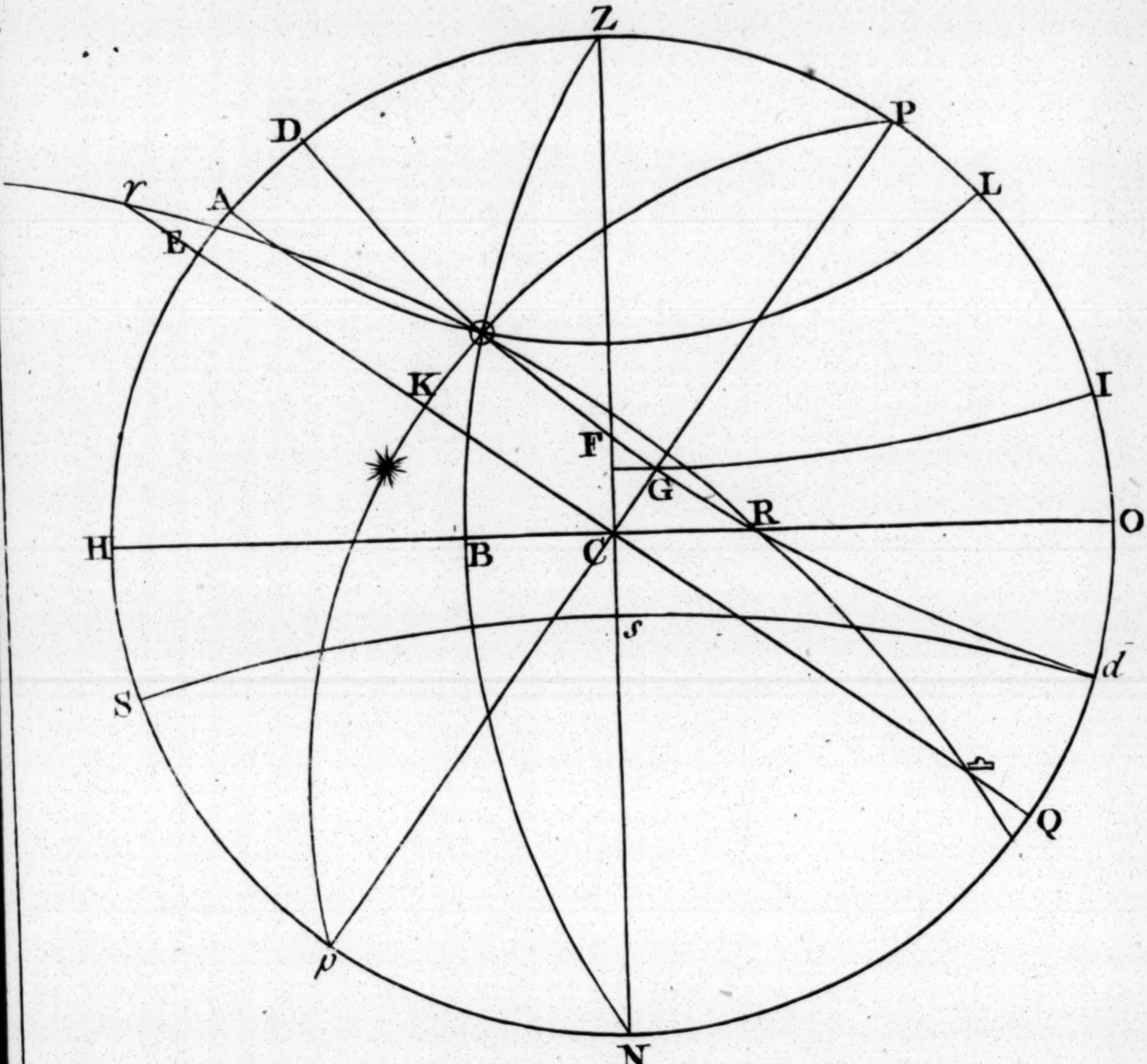


Projection.

X. p. 50.

6. *Horizon*, is a circle dividing the upper from the lower hemisphere, as HO, being 90° distant from the Zenith and Nadir. Fig. 52. 53. 55.

7. *Vertical Circles*, are circles passing thro' the Zenith and Nadir, Z ⊙ N.

8. *Circles of Longitude* in the heavens, pass thro' the poles of the ecliptic and cut it at right angles.

9. *Meridian of a Place*, is that Meridian which passes thro' the Zenith, as PZH.

10. *Prime Vertical*, is that which passes thro' the east and west points of the horizon.

III. *Lesser circles.*

1. *Parallels of Latitude* are parallel to the equinoctial on the earth, *parallels of altitude* are parallel to the horizon, *parallels of declination* are parallel to the equinoctial in the heavens.

2. *Tropics*, are 2 circles distant 23° 30′ from the equinoctial, the tropic of *Cancer* towards the north, the tropic of *Capricorn* towards the south.

3. *Polar Circles*, are distant 23° 30′ from the poles of the world, the *Arctic* circle towards the north, the *Antarctic* towards the south.

IV. *Angles and Arches of Circles.*

1. *Sun's* (or *Star's*) *Altitude*, is an arch of the Azimuth between the sun and horizon, as ⊙ B.

2. *Amplitude* is an arch of the horizon, between sun-rising and the east, or sun-setting and the west.

3. *Azimuth*, is an arch of the horizon between the sun's Azimuth circle, and the north or south, as HB, or OB; or it is the angle at the zenith, HZB or OZB.

4. *Right Ascension* is an arch of the equator between the sun's meridian, and the first point of Aries, as ♈ K.

5. *Ascensional Difference* is an arch of the equinoctial, between the sun's meridian, and that point

Fig. of the equinoctial that riſes with him, or it is the
52. angle at the pole between the ſun's and the ſix o'clock
53. meridian.

55. 6. *Oblique Aſcenſion* or *Deſcenſion*, is the ſum or difference of the right aſcenſion and the aſcenſional difference.

7. *Sun's Longitude*, is an arch of the ecliptic, between the ſun and firſt part of Aries, as ♈ ☉.

8. *Declination* is an arch of the meridian, between the equinoctial and the ſun, as ☉K.

9. *Latitude of a Star*, is an arch of a circle of longitude between the ſtar and ecliptic.

10. *Latitude of a Plane*, in an arch of the meridian between the equinoctial and the place.

11. *Longitude* of a place on the earth is an arch of the equinoctial, between the firſt meridian (Iſle of *Ferro*), and the meridian of the place. And *diff. longitude*, is an arch of the equator, between the meridians of the two places, or the angle at the pole.

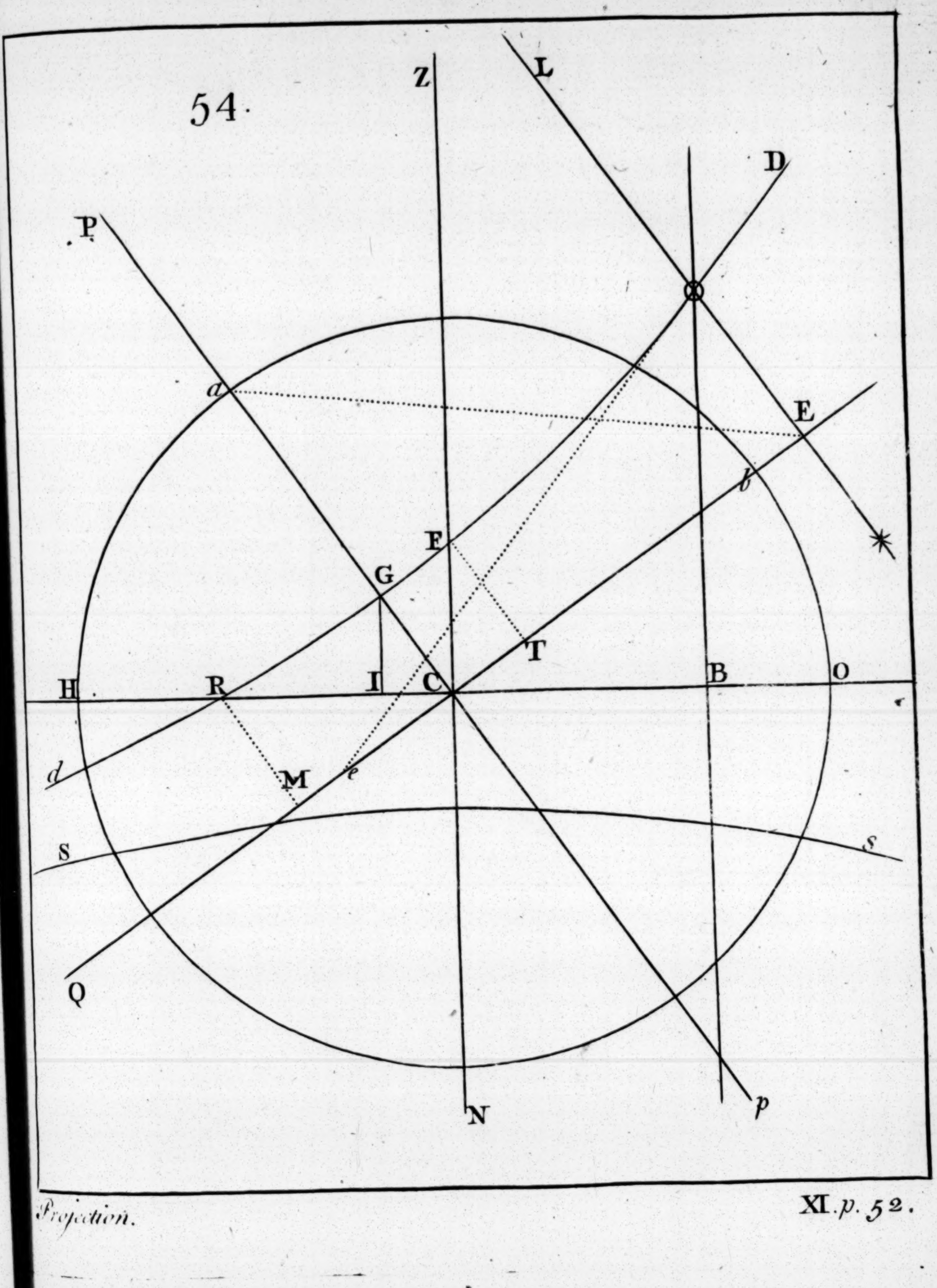
12. *Hour of the Day*, is an arch of the equinoctial, between the meridian of the place and the ſun's meridian, as EK; or it is the angle they make at the pole, as EPO.

Example I.

To project the ſphere upon the plane of the meridian, for May 12, 1767. *Latitude* 54° ½ *north, at a quarter paſt* 9 *o'clock before noon.*

I. *By the Orthographic Projection.*

52. Here we will project the convex ſide of the eaſtern hemiſphere. With the chord of 60° degrees deſcribe the primitive circle or meridian of the place HZON. Thro' the center C draw the horizon HO; ſet the latitude 54½ from O to P and from H to *p*, and draw P*p* the 6 o'clock meridian. Thro' C draw EQ perpendicular to P*p* for the equinoctial. Make ED, Q*d* 18° 5′ the declination May 12, and draw D*d* the ſun's parallel for that day. By Prop.



Projection.

Prop. XI. make ⊙ G ($3\frac{1}{4}$ hours or) 48° 45′ the Fig. 52.
sun's distance from the hour of 6, then ⊙ is the sun's place. Thro' ⊙ by Prop. V. draw AL parallel to H ⊙ for the sun's parallel of altitude. By Prop. VII. draw the meridian P ⊙ *p* and the azimuth Z ⊙ N. Also the ecliptic will be an ellipsis passing thro'. ⊙, which cannot conveniently be drawn in this projection. Also draw the parallel S*s* 18° below the horizon, and where it intersects D*d* is the point of day break, if there is any. Now the sun is at *d* at 12 o'clock at night, and rises at R, at 6 o'clock is at G, due east at F, at ⊙ a quarter past 9, and is at D in the meridian at 12 o'clock.

Draw GI parallel to HO. Then GR measured by Prop. X. is 27° 14′, and turned into time (allowing 15 degrees for an hour) shows how long the sun rises before 6, to be $1^h\ 49^m$; GI measured by Prop. X. gives the azimuth at 6, 79° 16′. CR by Cor. Prop. X. gives the amplitude 32° 19′, and CF gives his altitude when east 22° 25′. FG 13° 28′ (turned into time) is 54^m, and shews how long after 6 he is due east. IO is his altitude at 6, 14° 38′. AH 41° 53′ is his altitude at ⊙, or a quarter past 9; and ⊙L measured by Prop. X. is his azimuth from the north at the same time, 122° 40′. And thus the place of the moon or a star being given, it may be put into the projection, as at ✱. And its altitude, azimuth, amplitude, time of rising, &c. may all be found, as before for the sun.

II. *Stereographically.*

To project the sphere on the plane of the meridian, the projecting point in the western point of the horizon; with cord of 60, draw the primitive circle HZON, and thro' C draw HO for the horizon, and ZN perpendicular thereto for the prime vertical. Set the latitude from O to P, and from H to *p*, and draw P*p* the 6 o'clock meridian, and 53.

EQ

Fig. EQ perpendicular thereto for the equinoctial.
53. Make ED, Q*d* the declination, and by Prop. XII. draw DG*d*, the sun's parallel for the day. Draw the meridian P☉*p* by Prop. XVII. making an angle of 41° 15′ with the primitive, to intersect the sun's parallel in ☉, the sun's place at $9^h \frac{1}{4}$. Thro' ☉, by Prop. XII. draw the parallel of altitude A☉L; thro' ☉ draw, by Prop. XVII. the azimuth Z☉N. And by Prop. XII. draw the parallel S*sd* 18° below the horizon, if it cut R*d*, gives the point of day break. And thro' G draw the parallel of altitude GI. Lastly, by Prop. XX. thro' ☉ draw the great circle ♈☉♎ cutting the equinoctial EQ at an angle of 23° : 30′, and this is the ecliptic, ♈ the first point of Aries, and ♎ that of Libra.

This done, *d*R measured by Prop. XXIII. is 62° 46′, shows the time of sun rising; CR by Prop. XXII. is the amplitude 32° 19′. GI 79° 16′ by Prop. XXIII. the sun's azimuth at 6. IO 14° 38′ his altitude at 6. CF 22° 25′ by Prop. XXII. his altitude when east. GF 13° 28′ the time when he is due east. ☉B 41° 53′ by Prop. XXII. his altitude at a quarter past 9; the ∠☉ZP 122° 40′ by Prop. XXIV. his azimuth at that time. Also ♈☉, by Prop. XXII. is his longitude 51° 7′. ♈K his right ascension, 48° 40′.

And the place of the moon or a star being given, it may be put into the scheme as at ✱; and its time of rising, amplitude, azimuth, &c. found as before.

III. *Gnomonically.*

54. To project the eastern hemisphere upon a plane parallel to the meridian. About the center of projection C describe the circle HON with the tangent of 45 the radius of projection, for the primitive. Thro' C draw the horizon HO, and the prime vertical

tical ZN perpendicular thereto. Set the latitude 54 ½ from H to *a*, and draw the 6 o'clock meridian P*p*, and the equinoctial EQ perpendicular to it. Set the tangent of 48° 45′ (equal to 3¼ hours) from C to E, and by Prop. X. draw the meridian EL parallel to P*p*. Make E*e* = E*a*, and < E*e* ⊙ = 18° 5′ the sun's declination, then by Prop. XI. ⊙ is the sun's place. Thro' ⊙ draw the hyperbola D⊙*d* (by Prop. XIV.) for the sun's parallel of declination; and draw ⊙B perpendicular to HO, for his azimuth circle. And draw GI perpendicular to HO, and RM, FT, ∥ P*p*. Also the ecliptic is a right line passing thro' ⊙, and cutting EQ at an angle of 23° 30′, which is difficult to draw in this projection. Fig. 54.

Also by Prop. XIV. Draw the parallel S*s* 18° below the horizon, and if it intersects D*d*, it gives the point of sun rise.

Then if by Prop. XVII. or XI. you measure GR or rather CM, 27° 14′, you have the time of sun rising; GF or CT 13° 28, the time when he is due east. Also by Prop. XI. if you measure CR you have the amplitude 32° 19′. CI the comp. of his azimuth at six, 10° 44′. IG by Prop. XII. his altitude at 6, 14° 38′. CF his altitude when east, 22° 25′. And by Prop. XI. ⊙B = 41° 53′, his altitude a quarter past 9. CB the complement of his azimuth at that time, 32° 40′.

And the place of the moon or a star being given, its place in the projection may be determined as before, and all the requisites found.

Ex. 2.

To project the sphere upon the plane of the solstitial colure for latitude 54½ *N. May* 23, 1767, *at* 10 *o'clock in the morning.*

Stereogra-

Fig. 55.

Stereogrophically.

The projection of the western hemisphere, the first point of Libra, the projecting point. Describe the solstitial colure PEpQ, and the equinoctial colure Pp perpendicular to it; and thro' C draw the equinoctial EQ perpendicular to Pp. Set 23° 30 from E to ♋, and from Q to ♑, and draw the ecliptic ♋ ♑. Set the sun's longitude 61° 42′ from C to ☉, and thro' ☉ draw P☉Kp for the 10 o'clock meridian. Make KA (two hours or) 30°, and draw PAp for the meridian of the place. Set the latitude of the place $54\frac{1}{2}$ from A to Z, and Z is the zenith. About the pole Z describe the great circle BHS for the horizon of the place. Thro' Z and ☉ draw an azimuth circle Z☉B.

Then you have ☉K the sun's declination 20° 33′. CK his right ascension 59° 35′. ☉B his altitude at 10 o'clock 49° 10′; the ∠ AZ☉ or PZ☉ his azimuth at 10 = HB, 45° 44′. H the south point of the horizon. I the point of the ecliptic that is in the meridian. T the point of the ecliptic that is setting in the horizon.

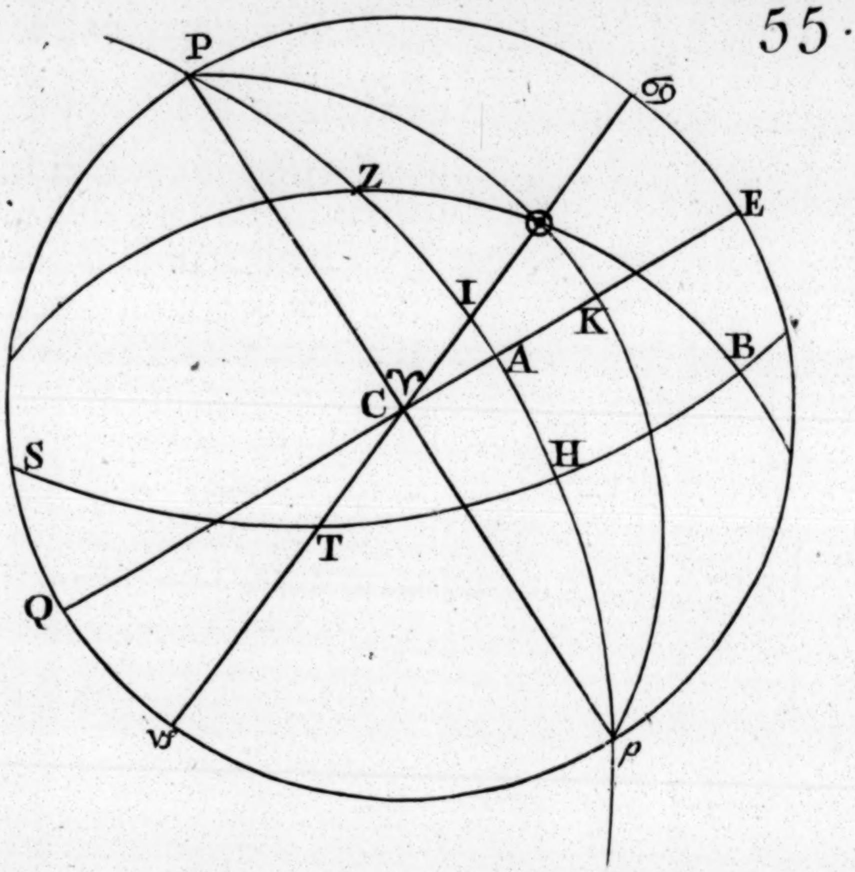
Example. 3.

To project the sphere on the plane of the horizon, Lat. $35\frac{1}{2}$, *N. July* 31, 1767, *at* 10 *o'clock.*

Gnomonically.

56. To project the upper hemisphere on a plane parallel to the horizon. With the radius of projection and center C, describe the primitive circle ADB. Thro' C draw the meridian PE, and AS perpendicular to it for the prime vertical. Set off CP $35\frac{1}{2}$ the latitude and P is the N. Pole, and perpendicular to CP draw Pp the 6 o'clock meridian. Set the complement of the latitude from C to E; and draw EQ perpendicular to CE for the equinoc-

55.



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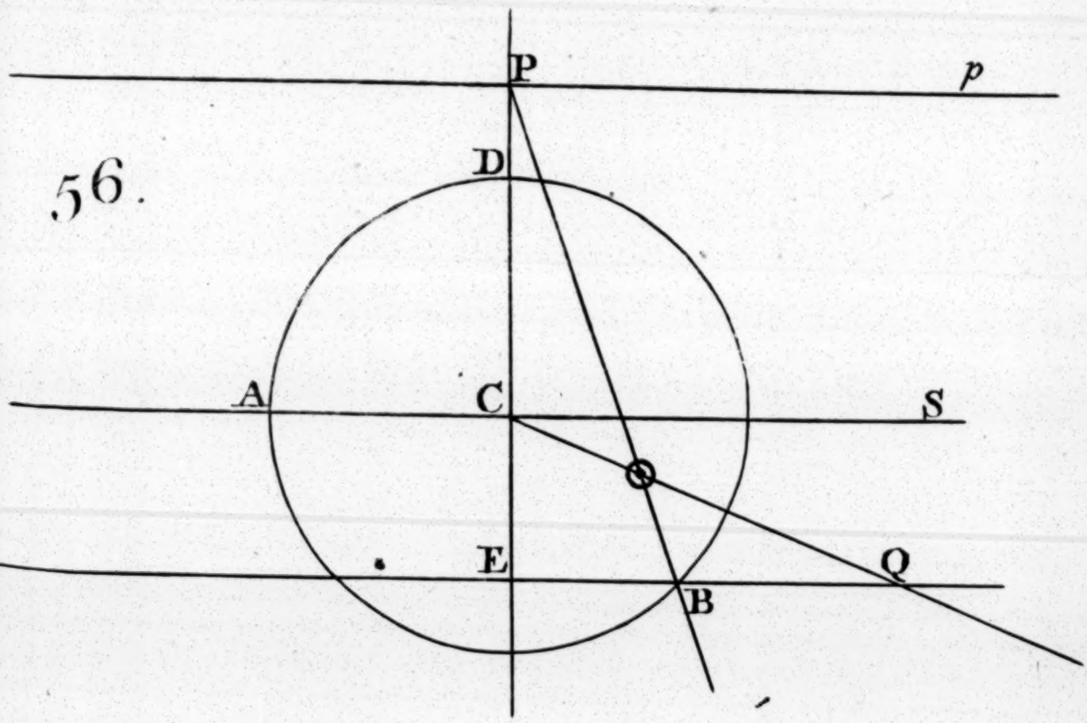

equinoctial. Make EB 30° (or 2 hours) and draw the 10 o'clock meridian PB. Set the sun's declination 18° 27′ from B to ⊙. And ⊙ is the place of the sun at 10 o'clock. Thro' ⊙ draw the azimuth circle CQ; likewise thro' ⊙, a parallel to the equinoctial EQ may easily be described by Prop. XV. for the sun's parallel that day. Fig. 56.

Then C ⊙ measured by Prop. XI. is 31° 30′ the complement of the altitude. And the angle EC⊙ measured by Cor. Prop. XII. is his azimuth, 65° 10.

Scholium.

After this manner may any Problems of the Sphere be solved by any of these Projections, or upon any planes, but upon some more commodiously than upon others. And if in a spherical triangle any sides or angles be required, they may be projected according to what is given therein, according to any of these kinds of projection before delivered; and it will be most easily done, when you chuse such a plane to project on, that some given side may be in the primitive, or a given angle at the center; and then you need draw no more lines or circles than what are immediately concerned in that Problem. But always chuse such a plane to project on, where the lines and circles are most easily drawn, and so that none of them run out of the scheme.

FINIS.

E R R A T A.

b signifies reckon from the bottom.

pag	line	read
5	1	fig. 2.
8	18	fig. 5.
9	3b	off the ſines,
13	17	fig. 12.
14	2b	the 2 laſt lines ſhould be indented and roman.
	21	C*p*A + CA*p* ;
15	3	the 3d line ſhould be indented, and the 3 following lines roman.
16	5	ECI = CAD
18	4	if, *p*, *q*, be
20	7	projection C,
21	15	$\frac{OG + OD}{2}$:
25	9b	points A, B, G,
29	6b	interſection *p*,
30	2	*g*C*t* required.
	11b	pole *p*, draw
33	17	of this circle
36	6b	TV, and
	2b	at *s* cut
38	2b	to *o*S ;
43	2	CL = radius
44	20	A to *d*,